Advanced code-based cryptography

Daniel J. Bernstein
University of Illinois at Chicago &
Technische Universiteit Eindhoven

Lattice-basis reduction

Define $L = (0, 24)\mathbf{Z} + (1, 17)\mathbf{Z}$
$= \{(b, 24a + 17b) : a, b \in \mathbf{Z}\}$.

What is the shortest nonzero vector in L?

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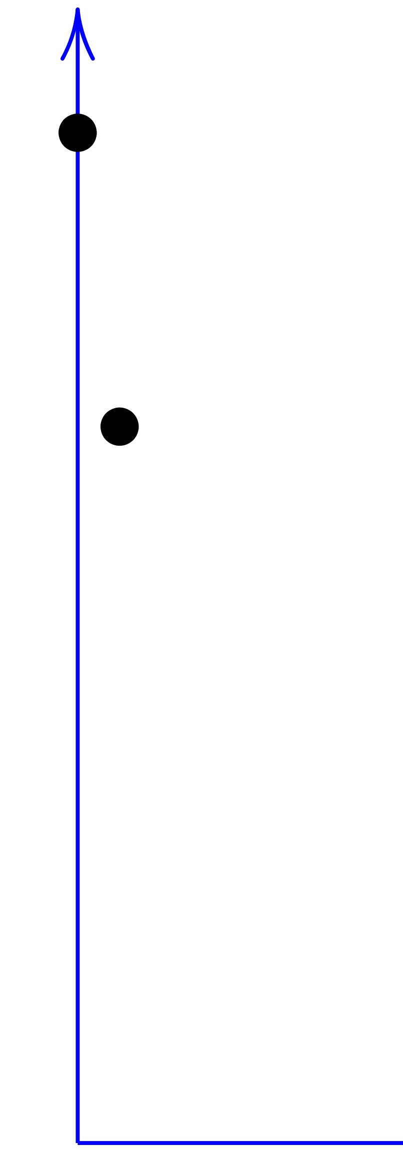

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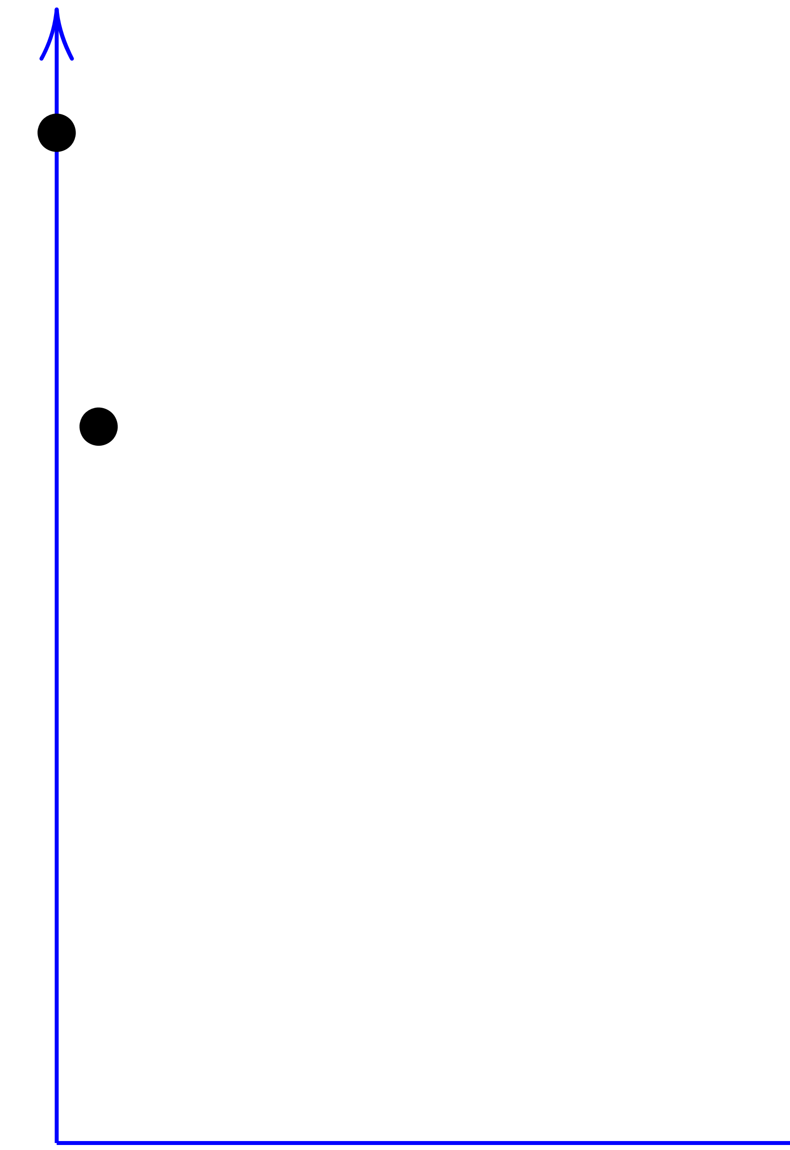

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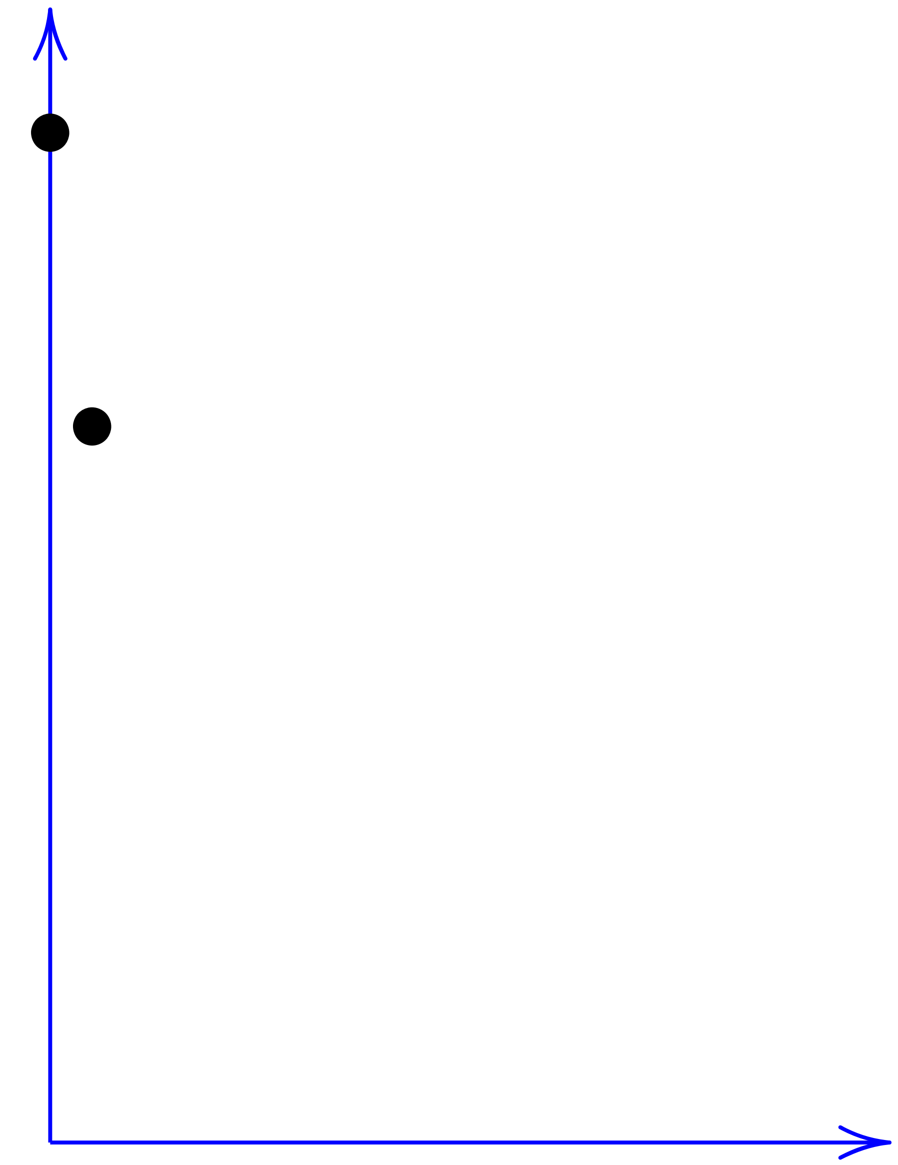

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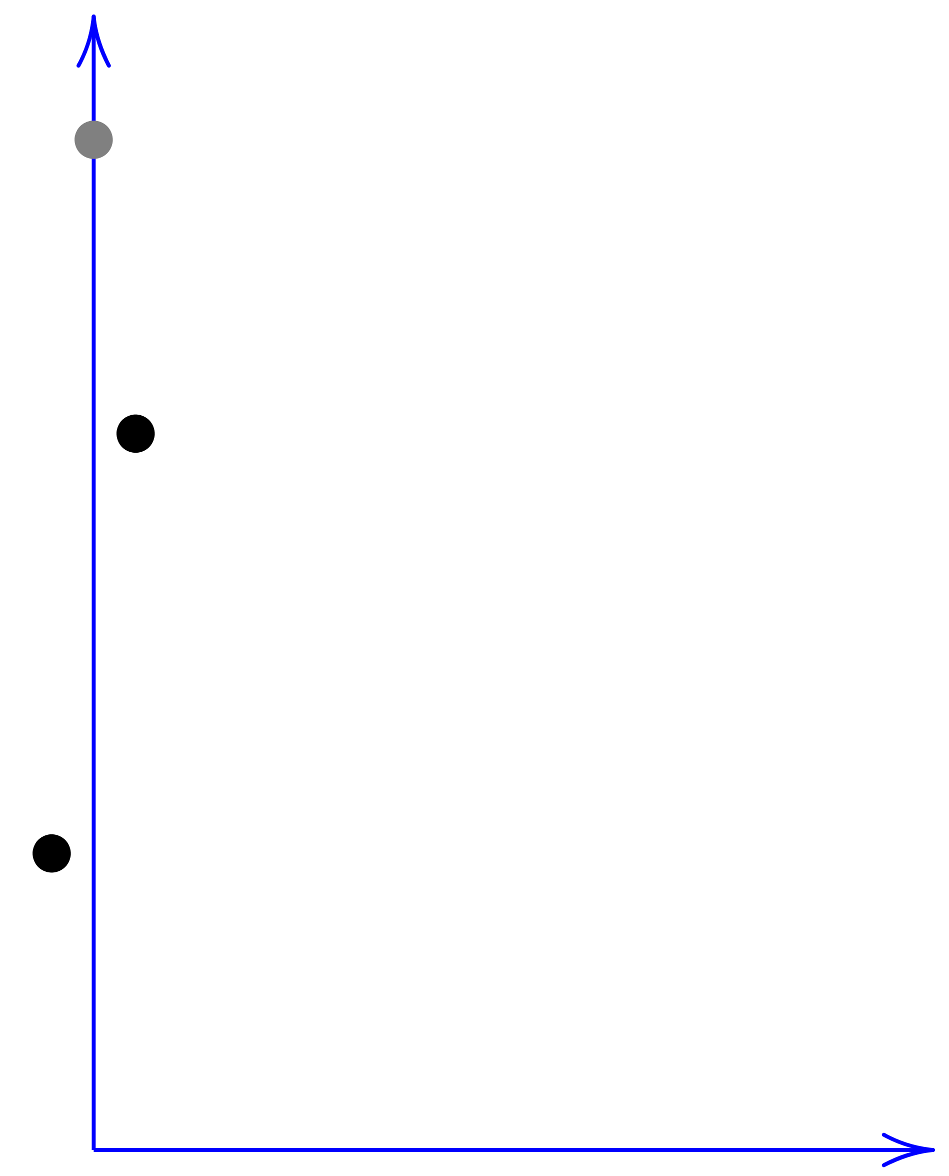

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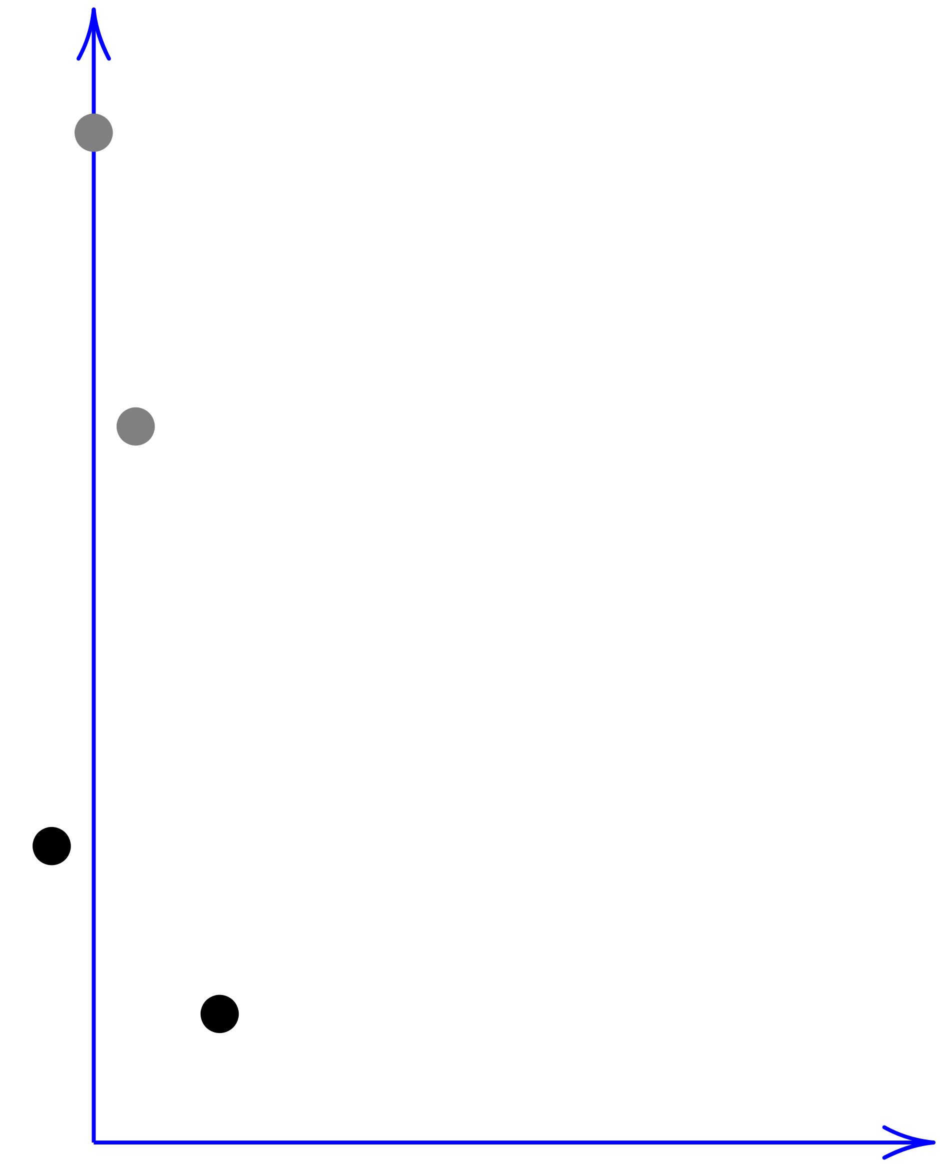

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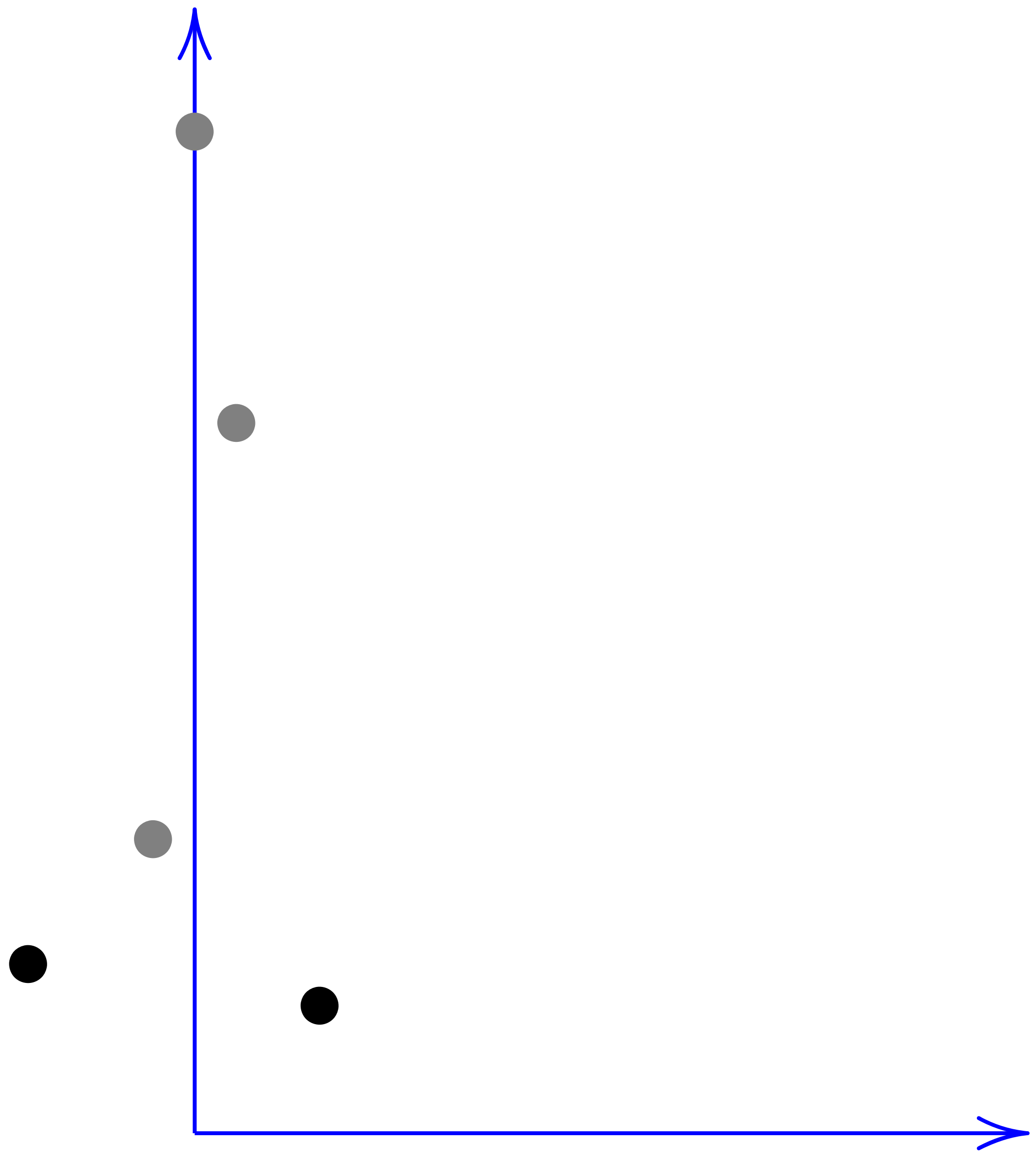

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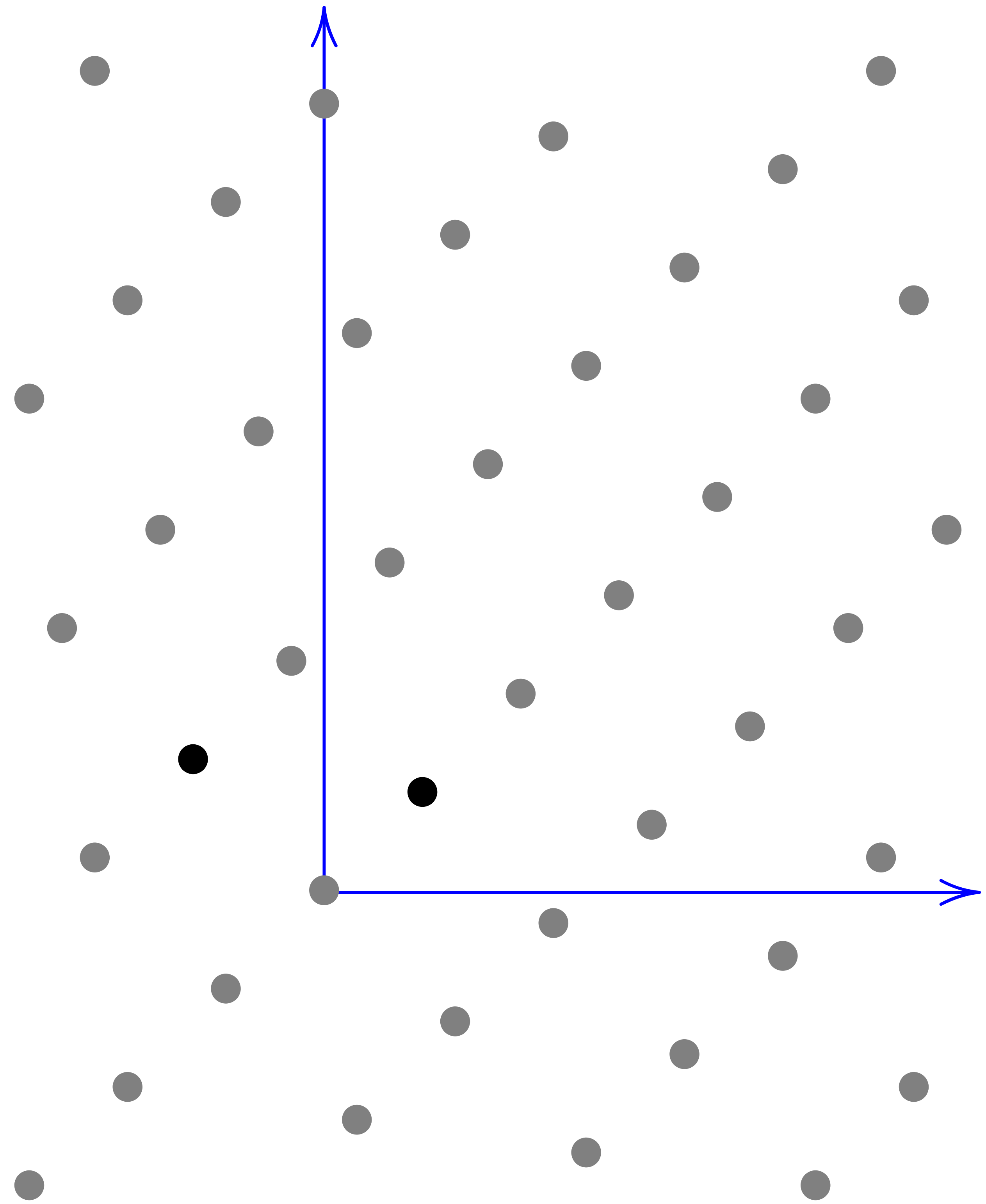

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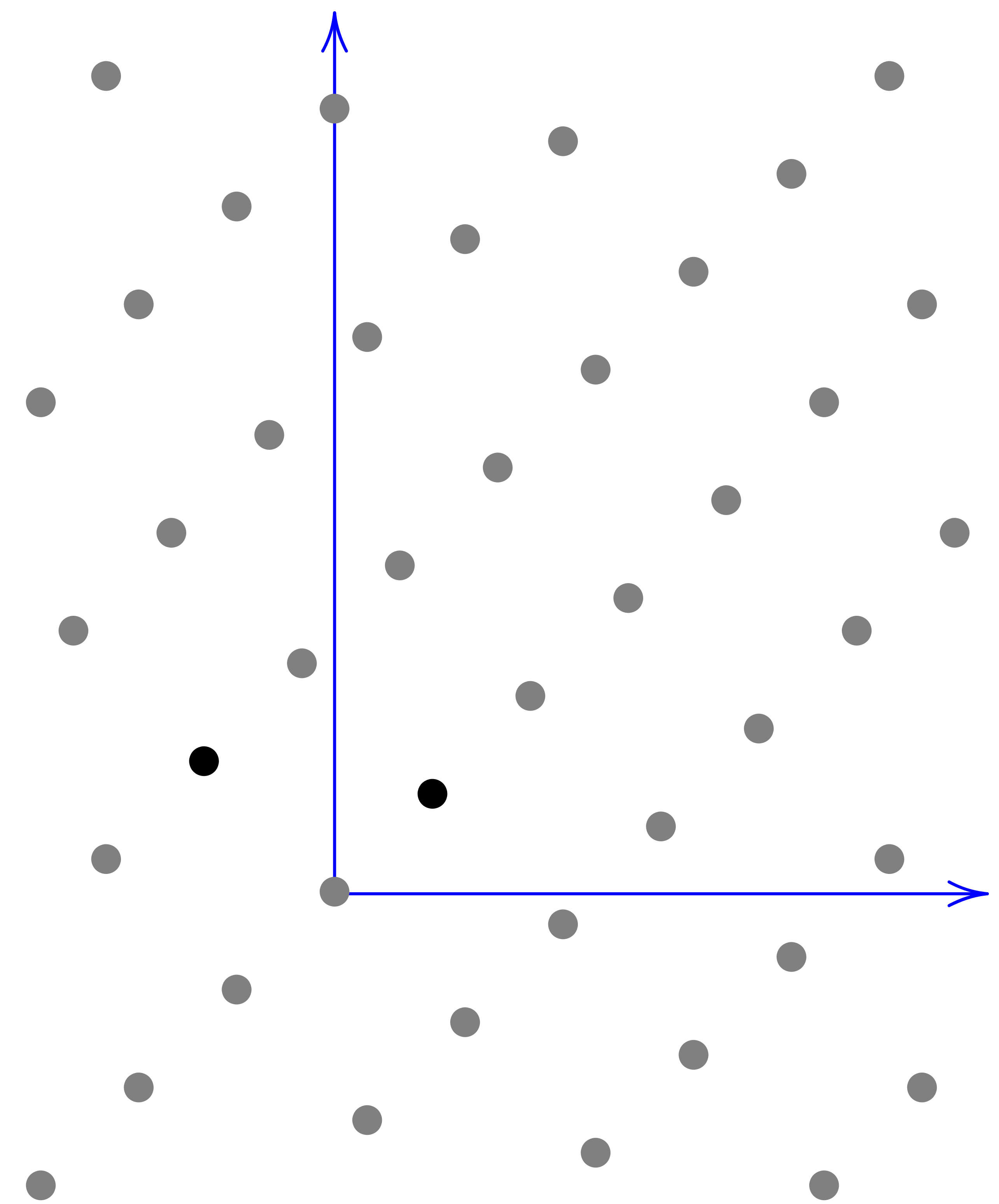

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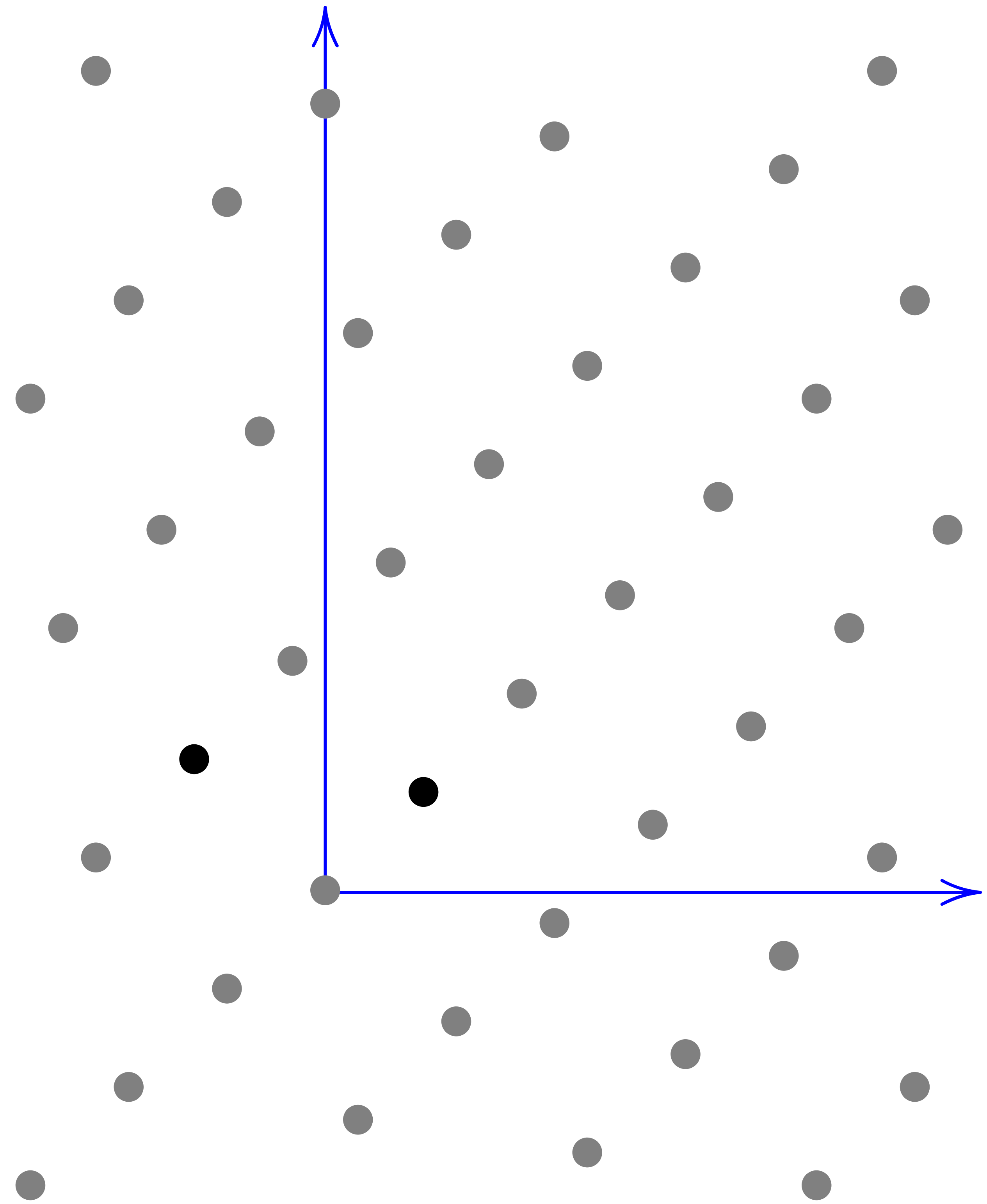

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Define $L = (0, 25)$

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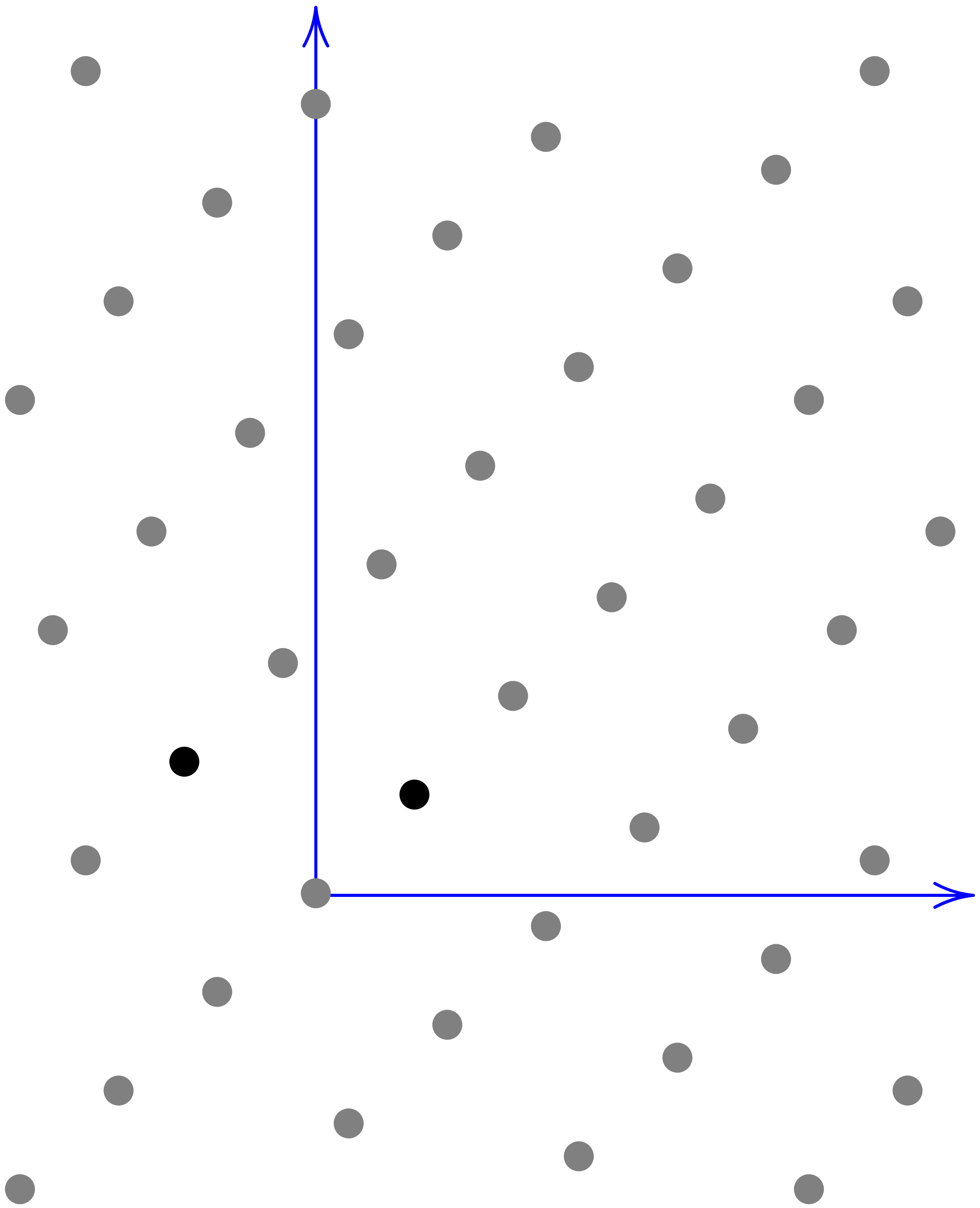

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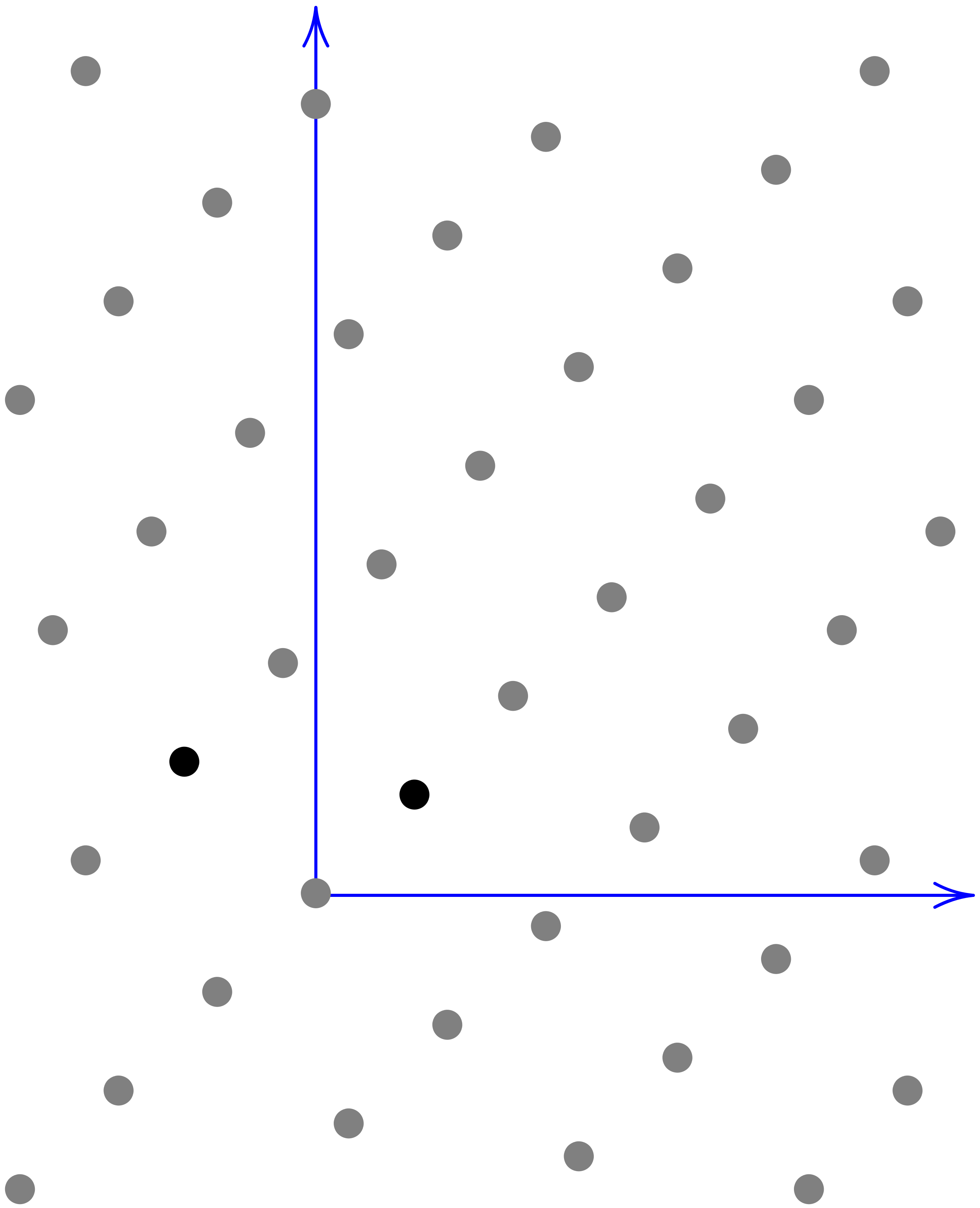

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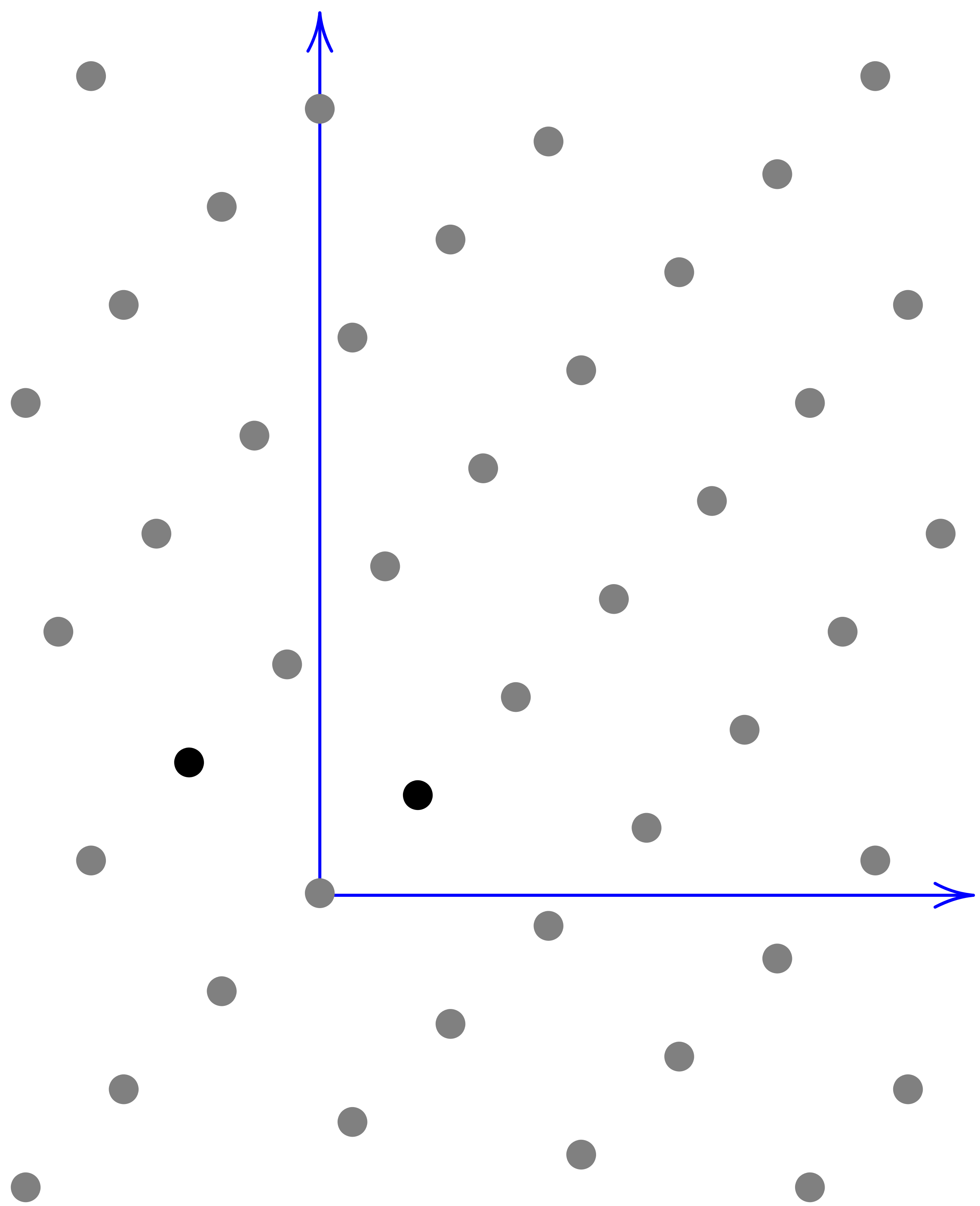

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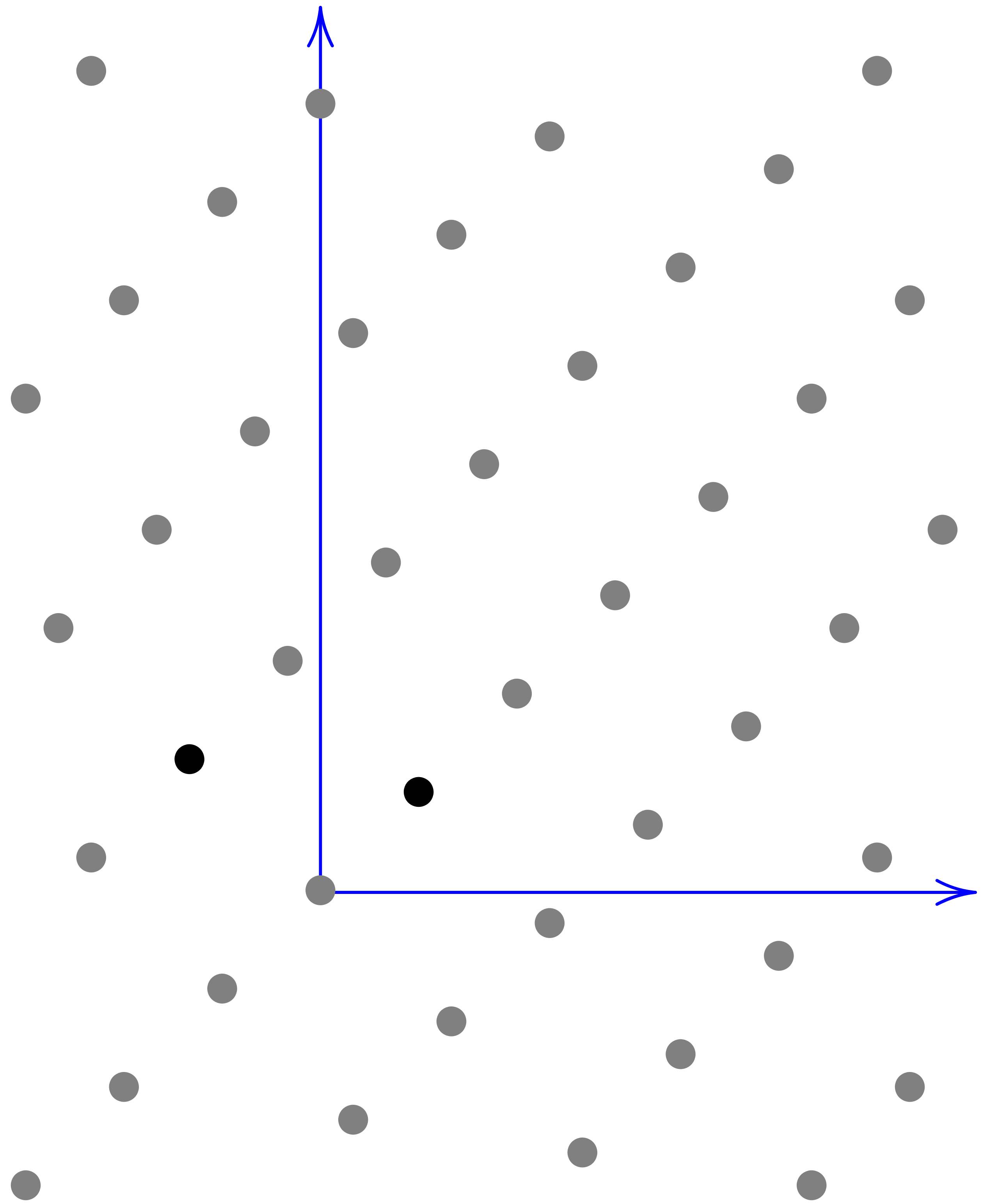

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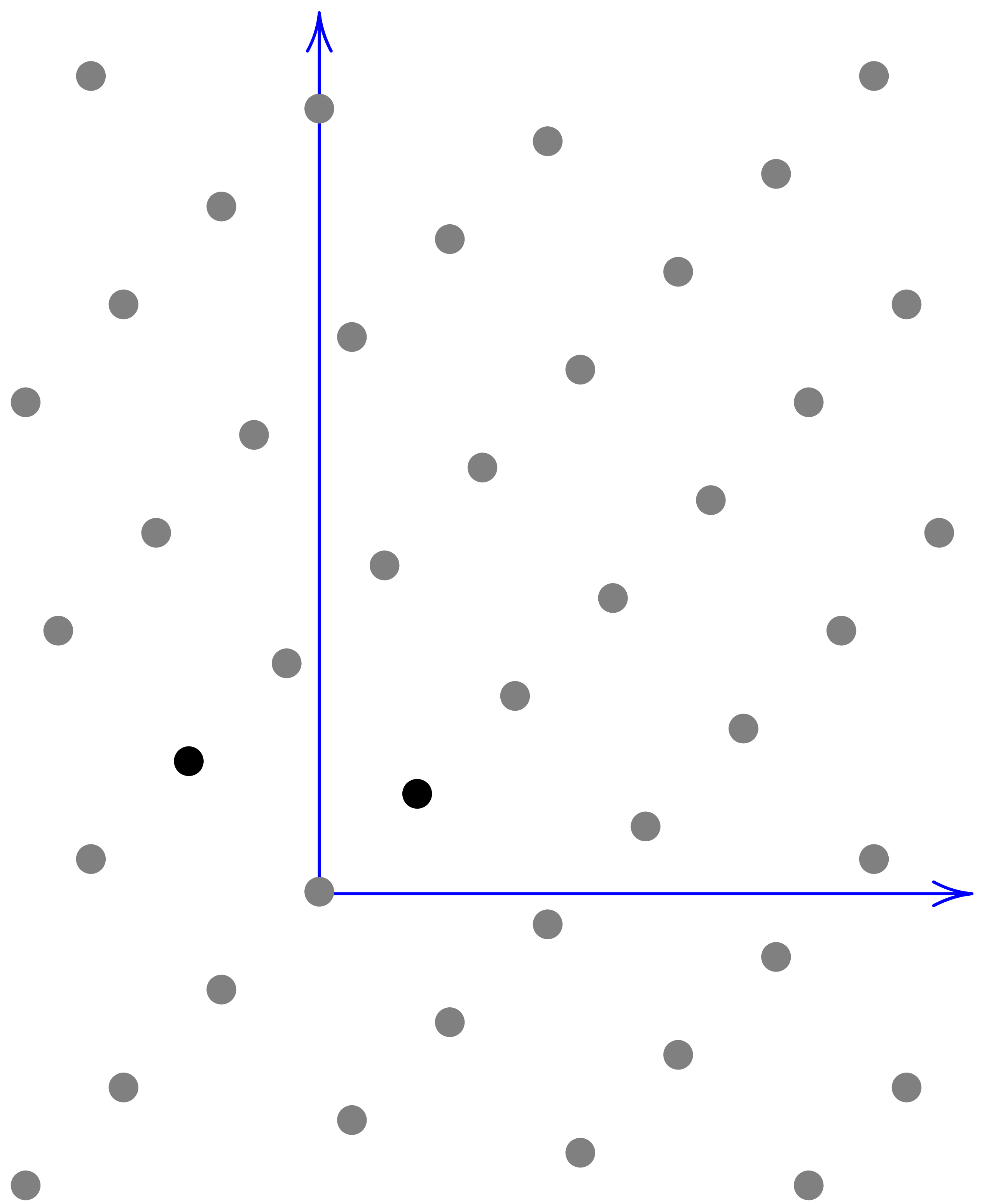

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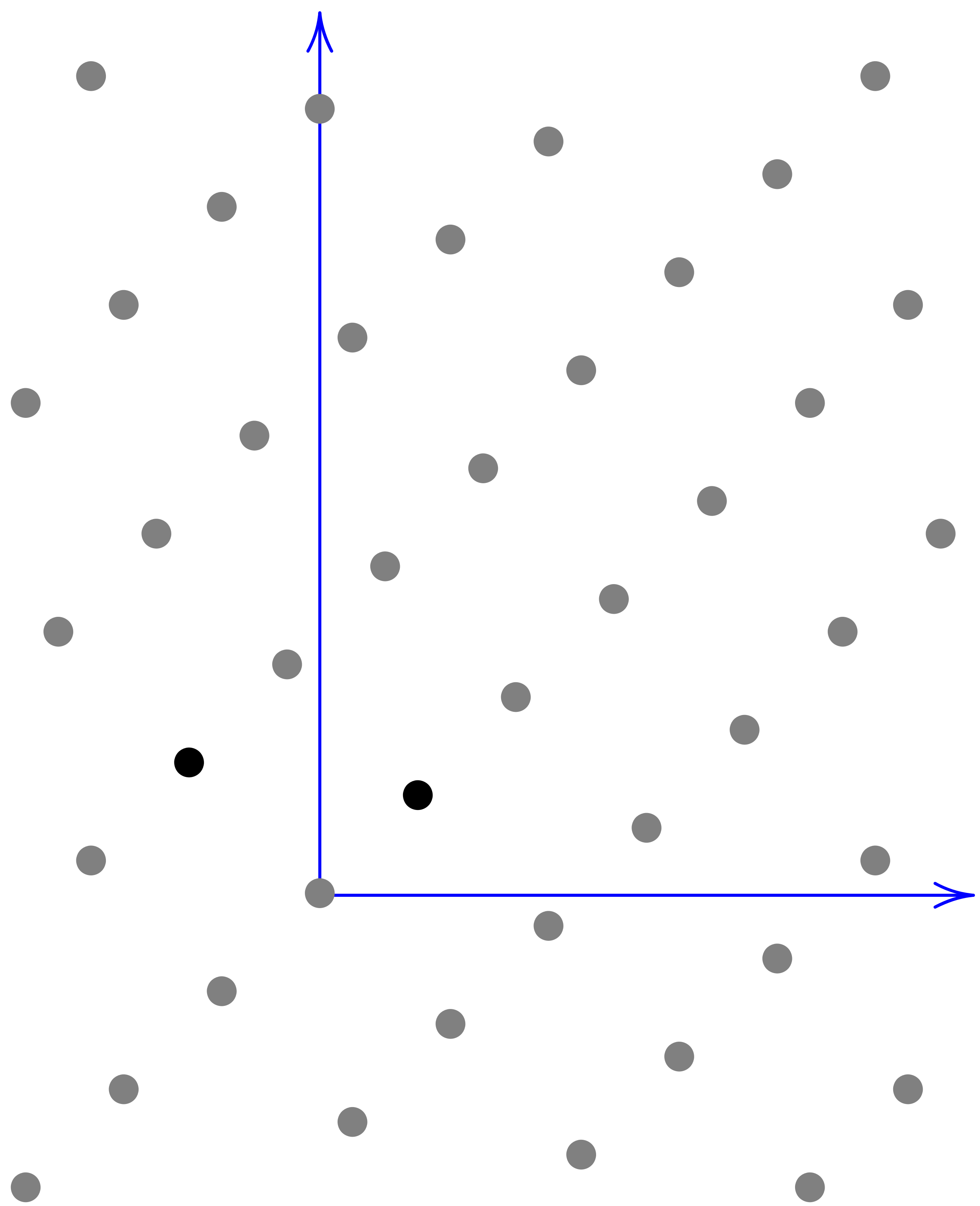

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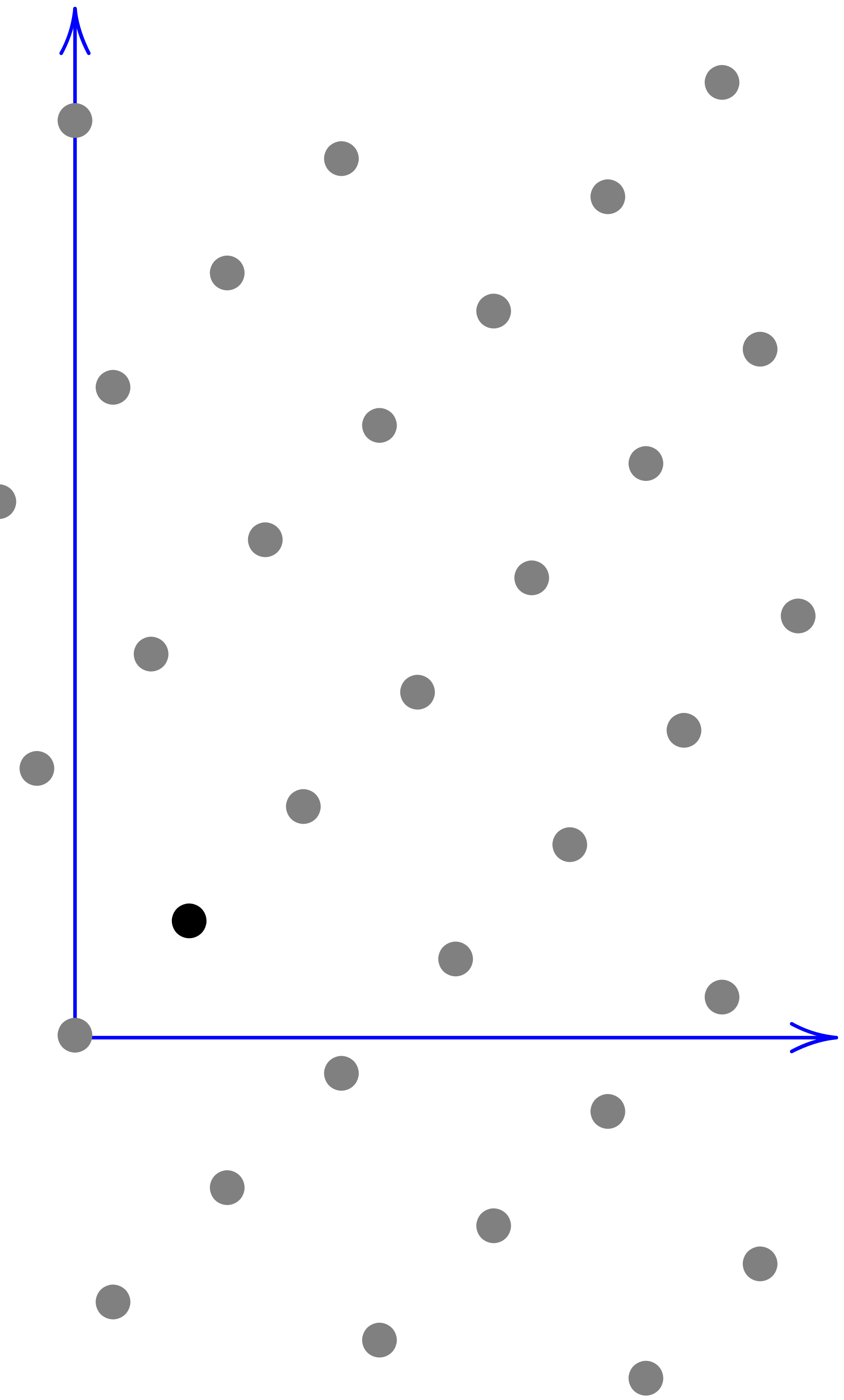

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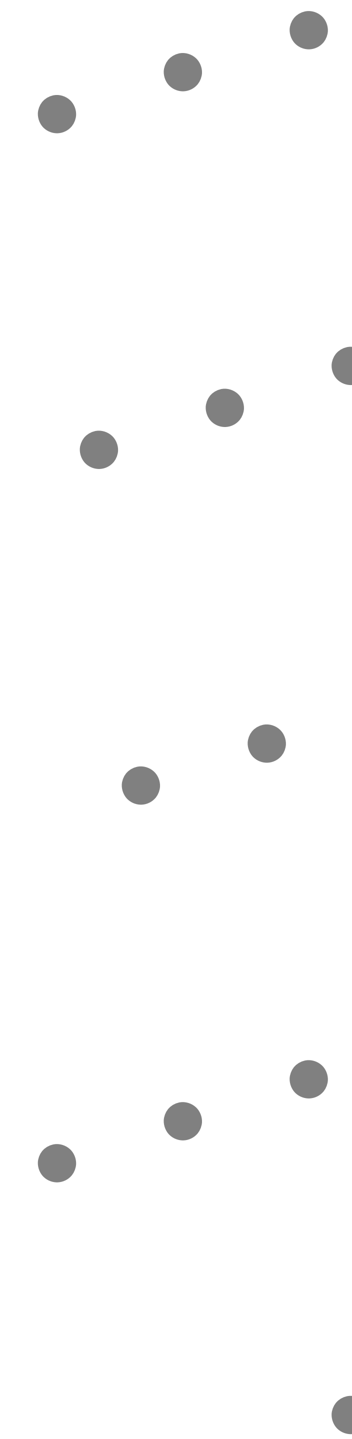

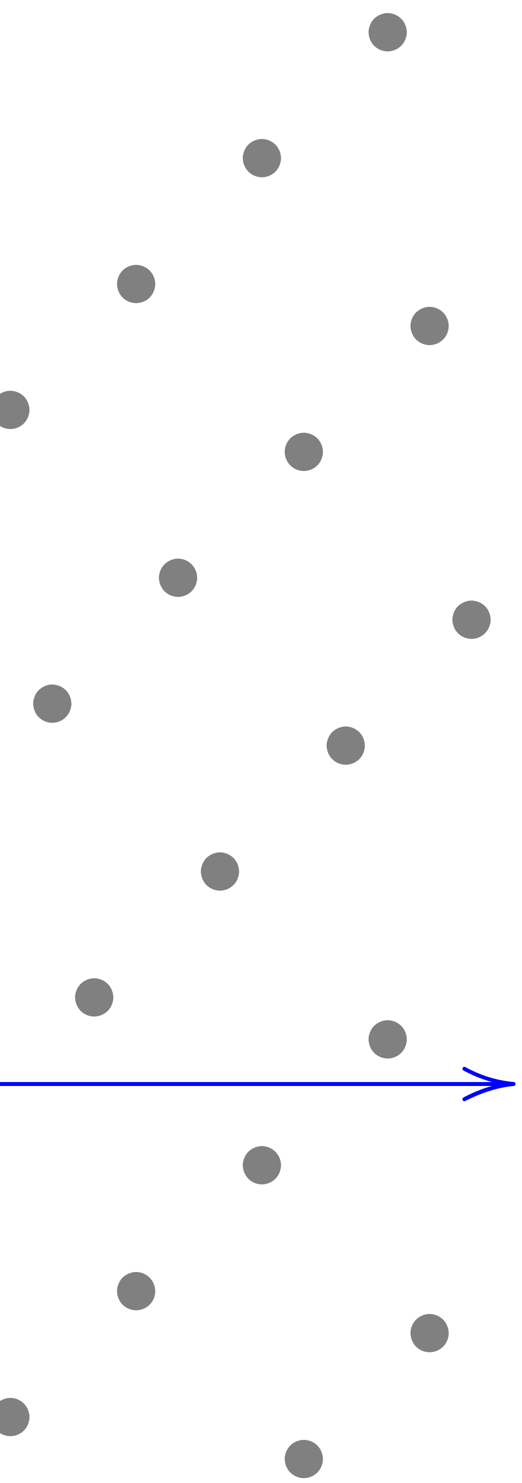

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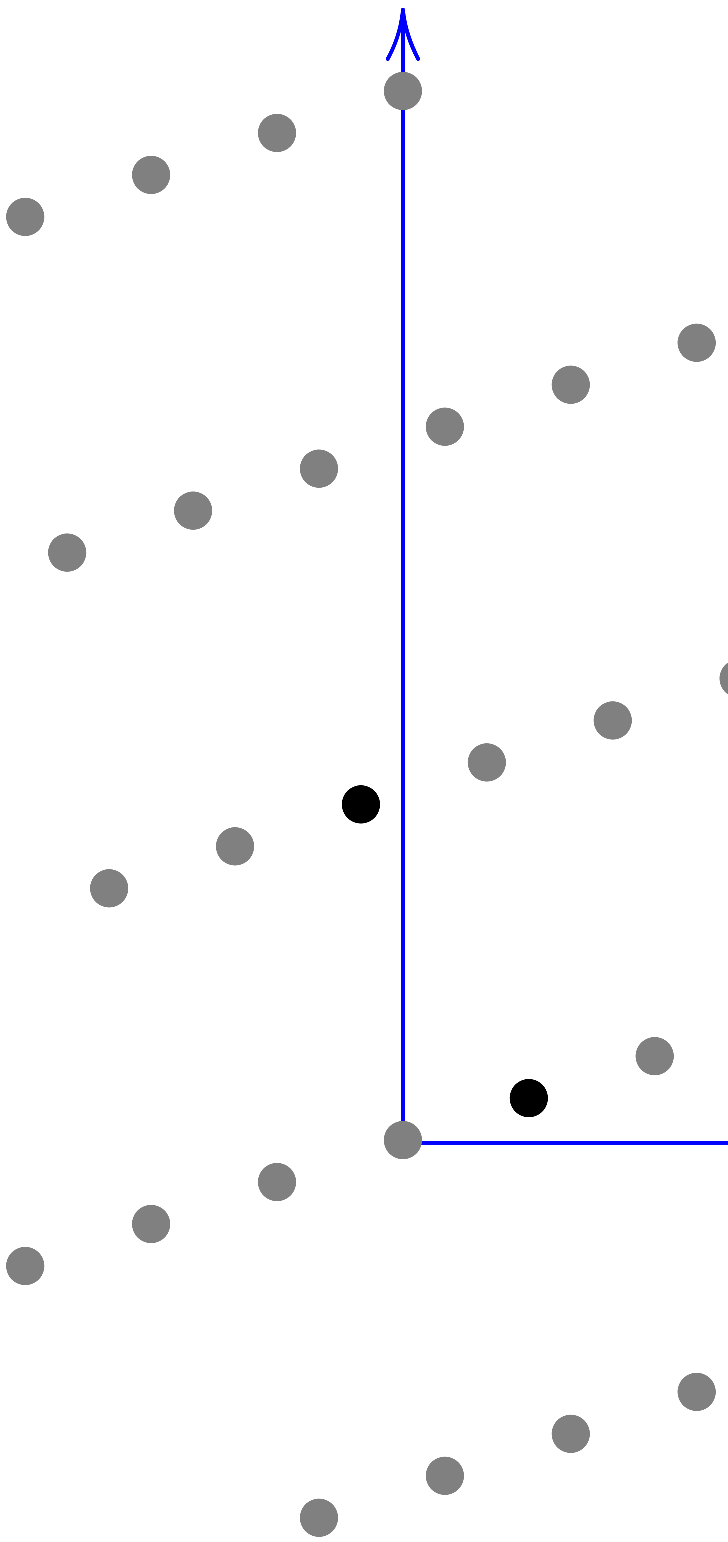

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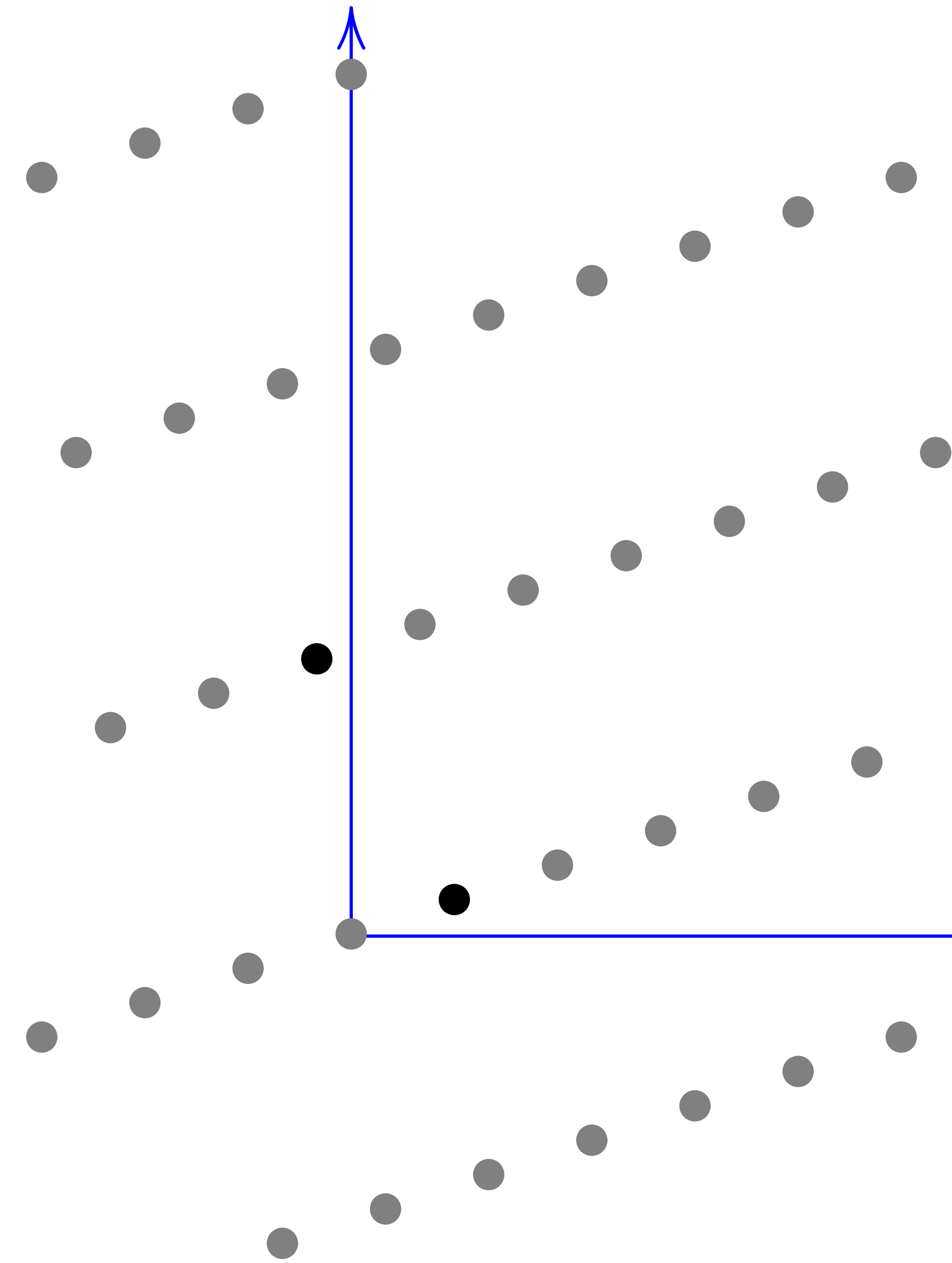

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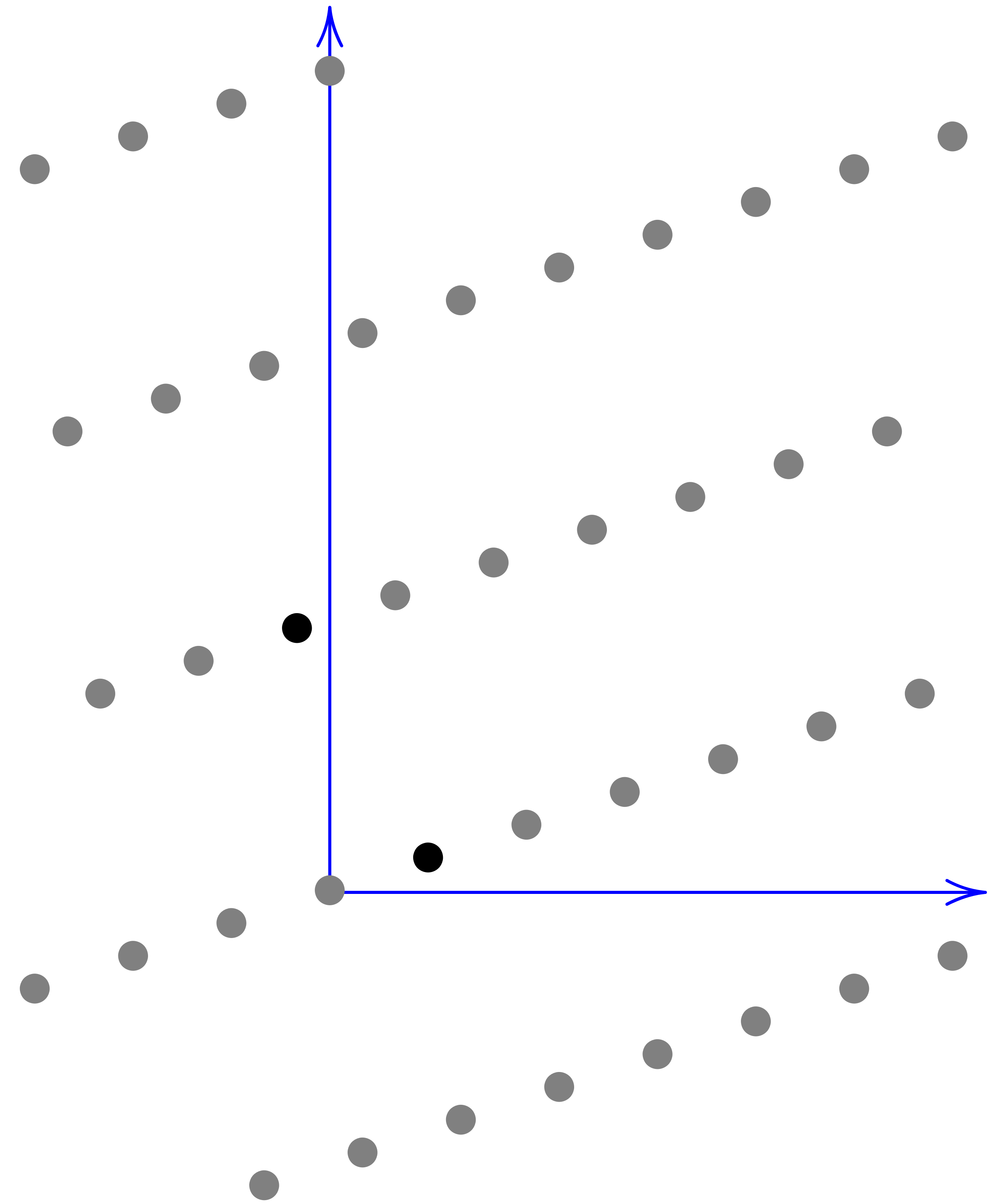

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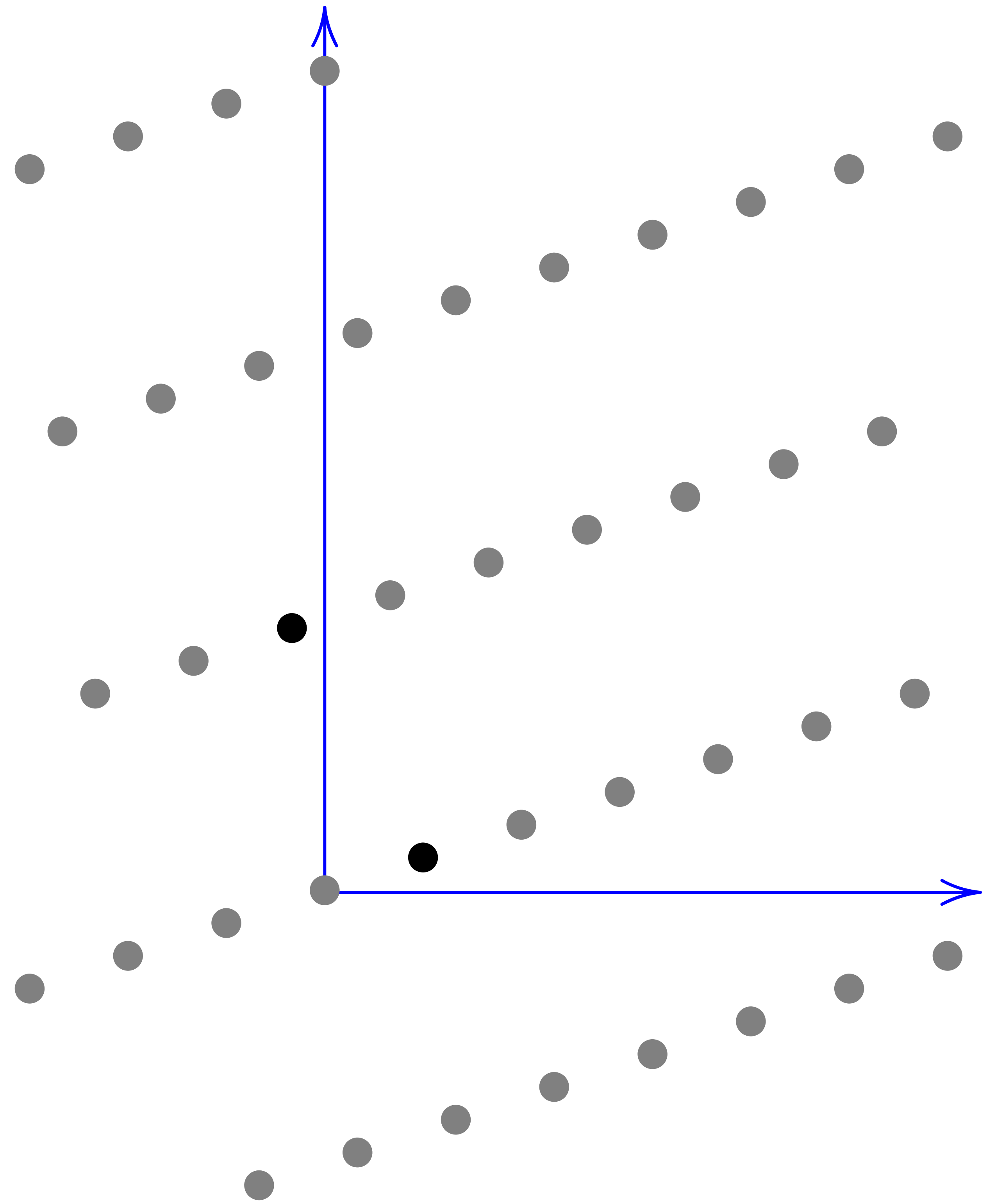

Polynom

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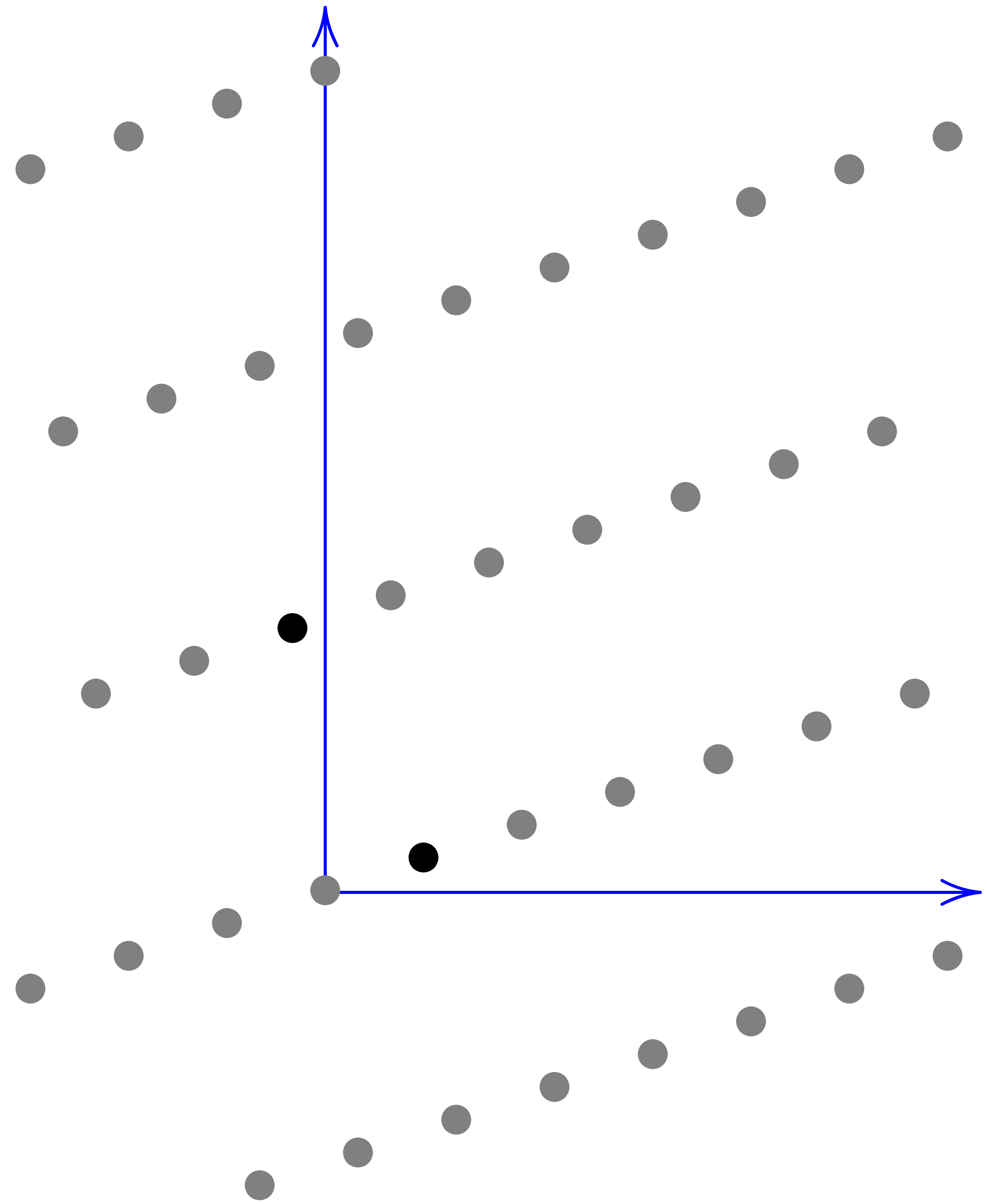

Polynomial lattices

Define $P = \mathbf{F}_2[x]$,

$r_0 = (101000)_x =$

$r_1 = (10011)_x = x$

$L = (0, r_0)P + (1,$

What is the shorte

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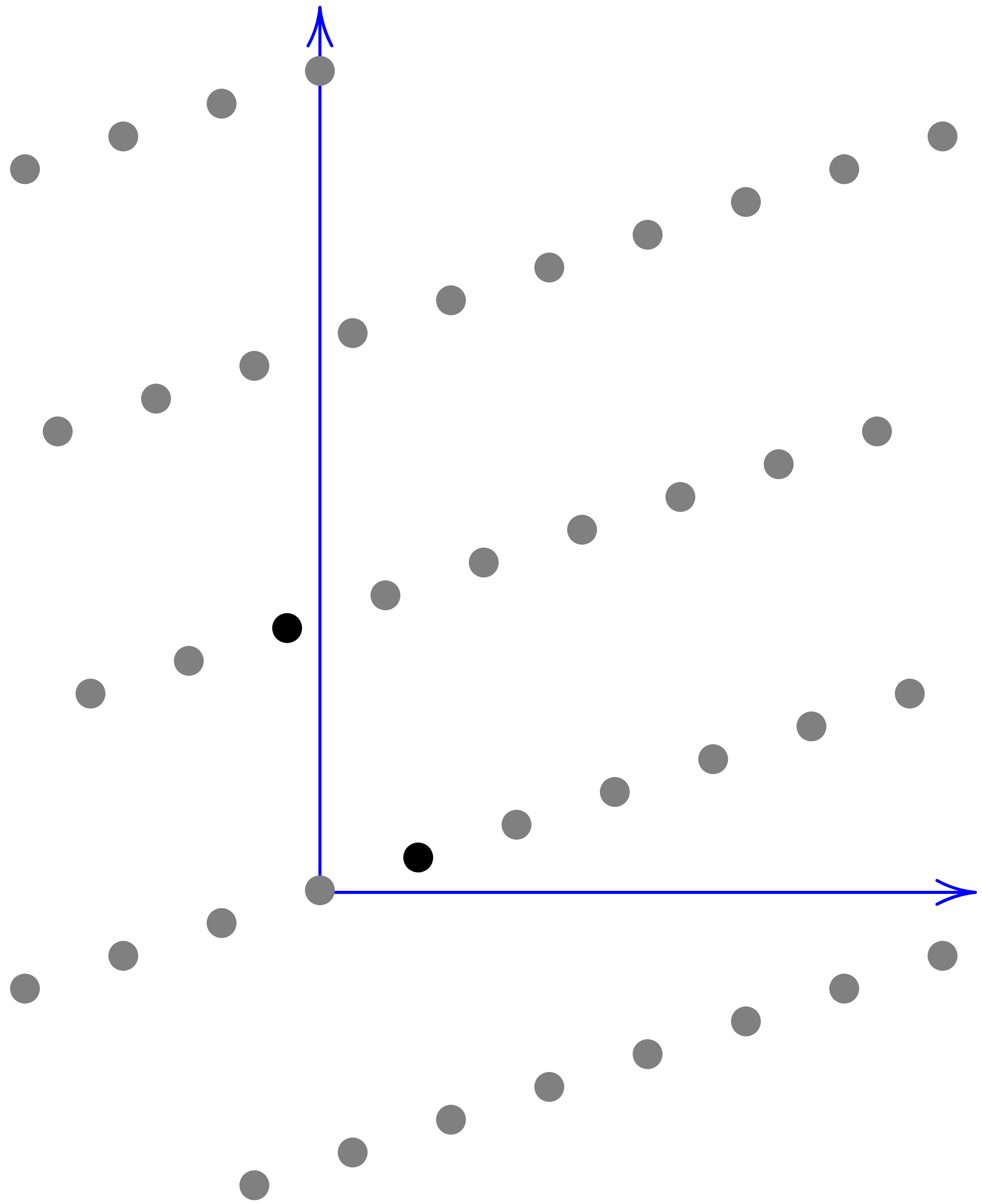

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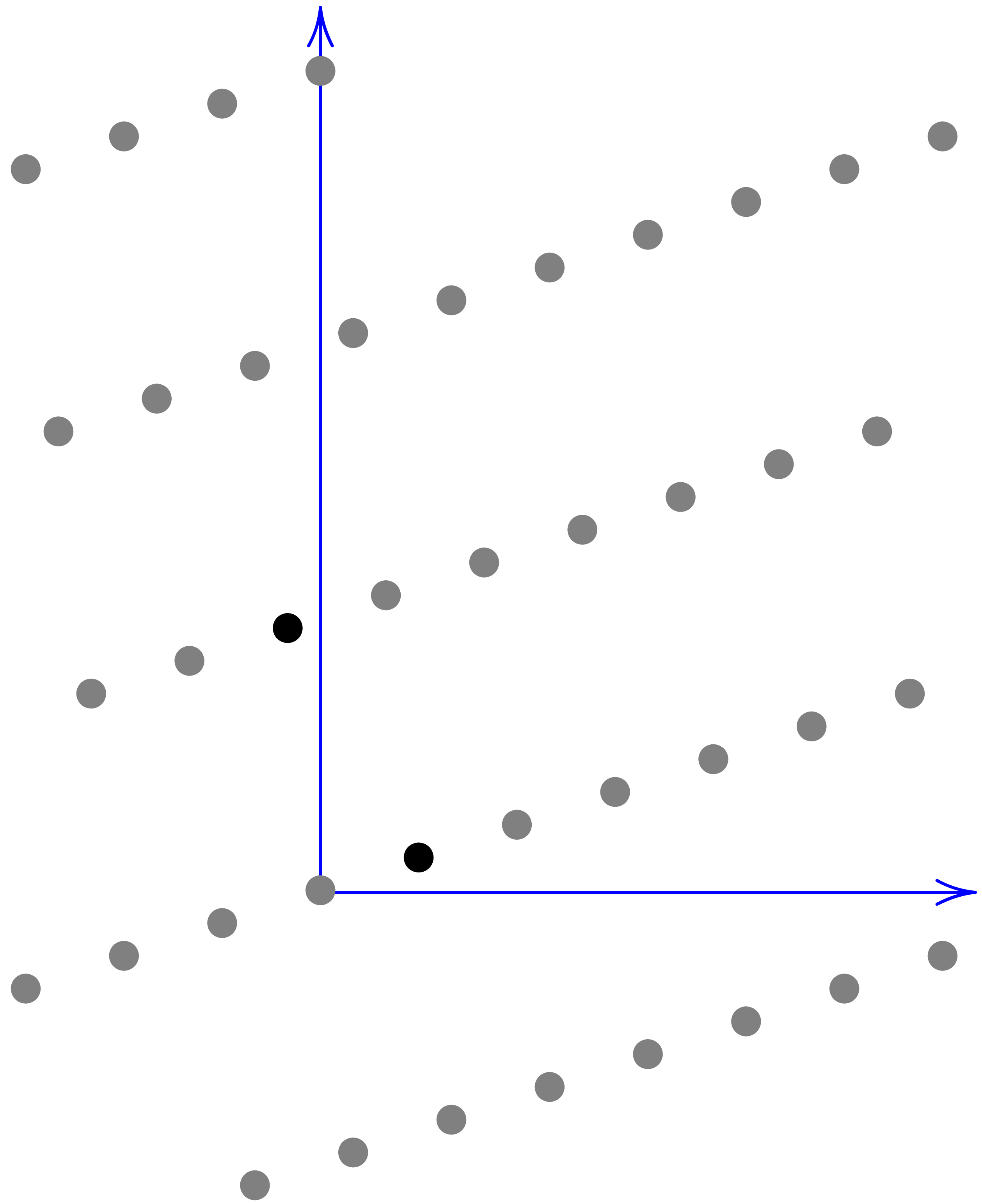

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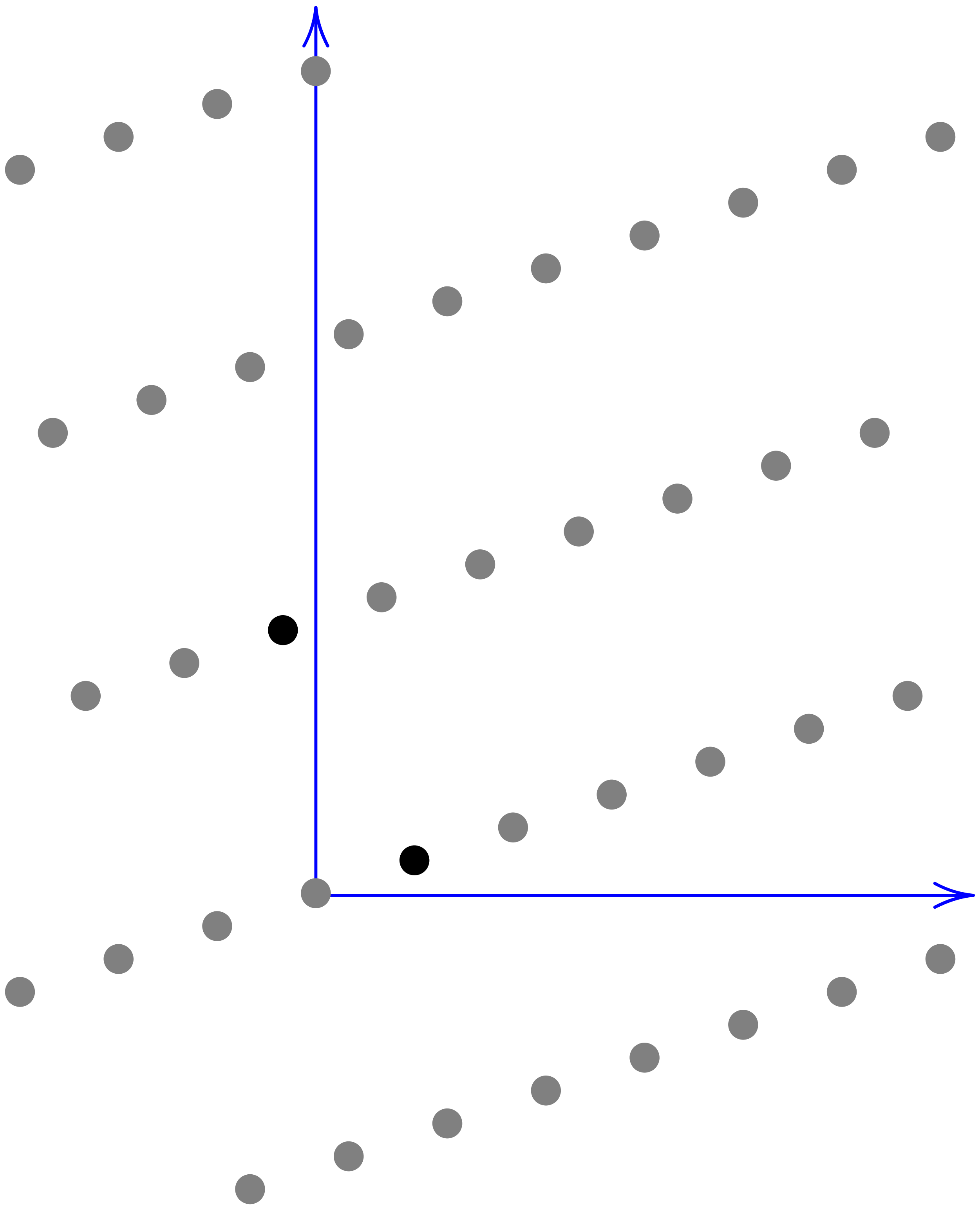

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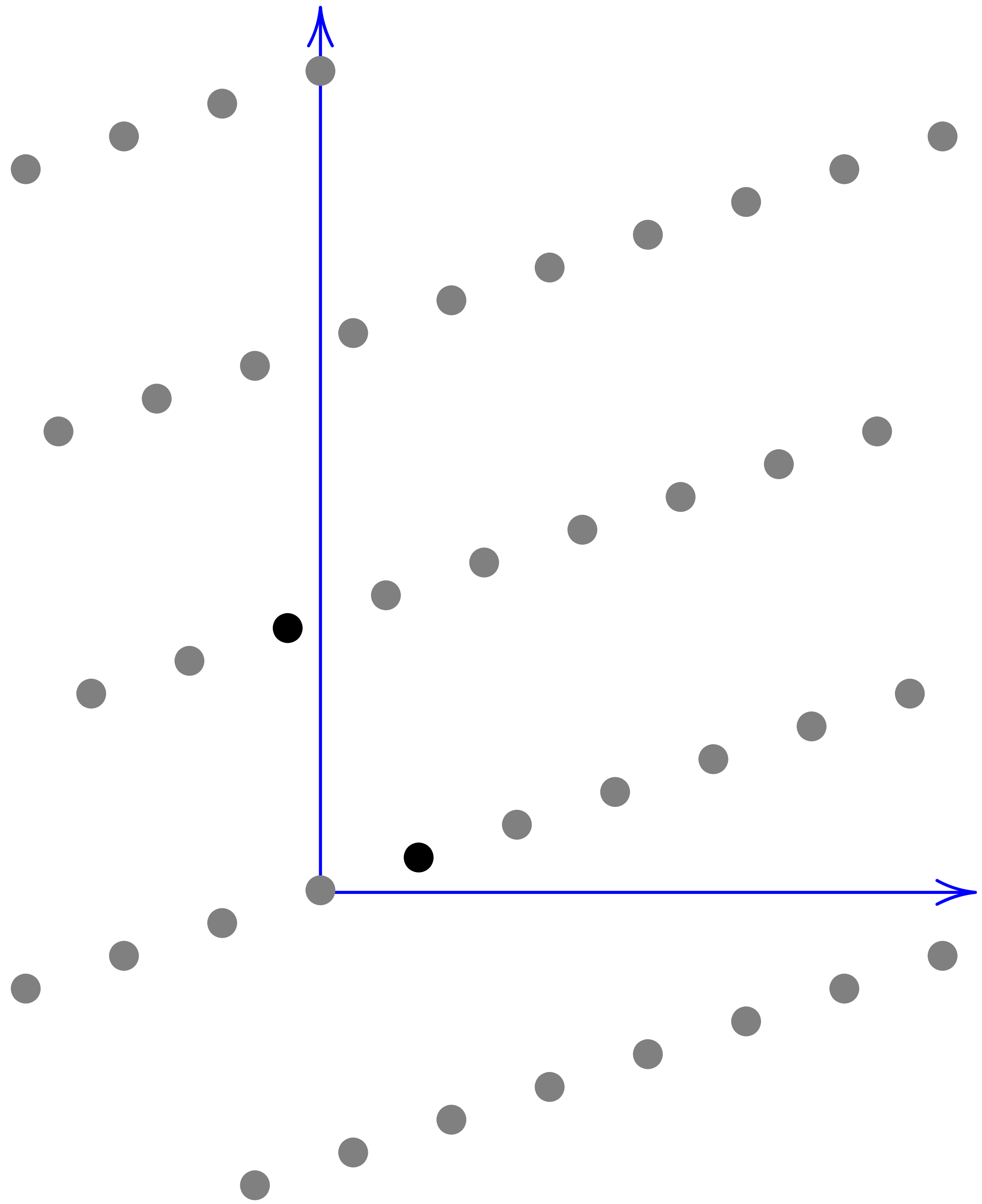

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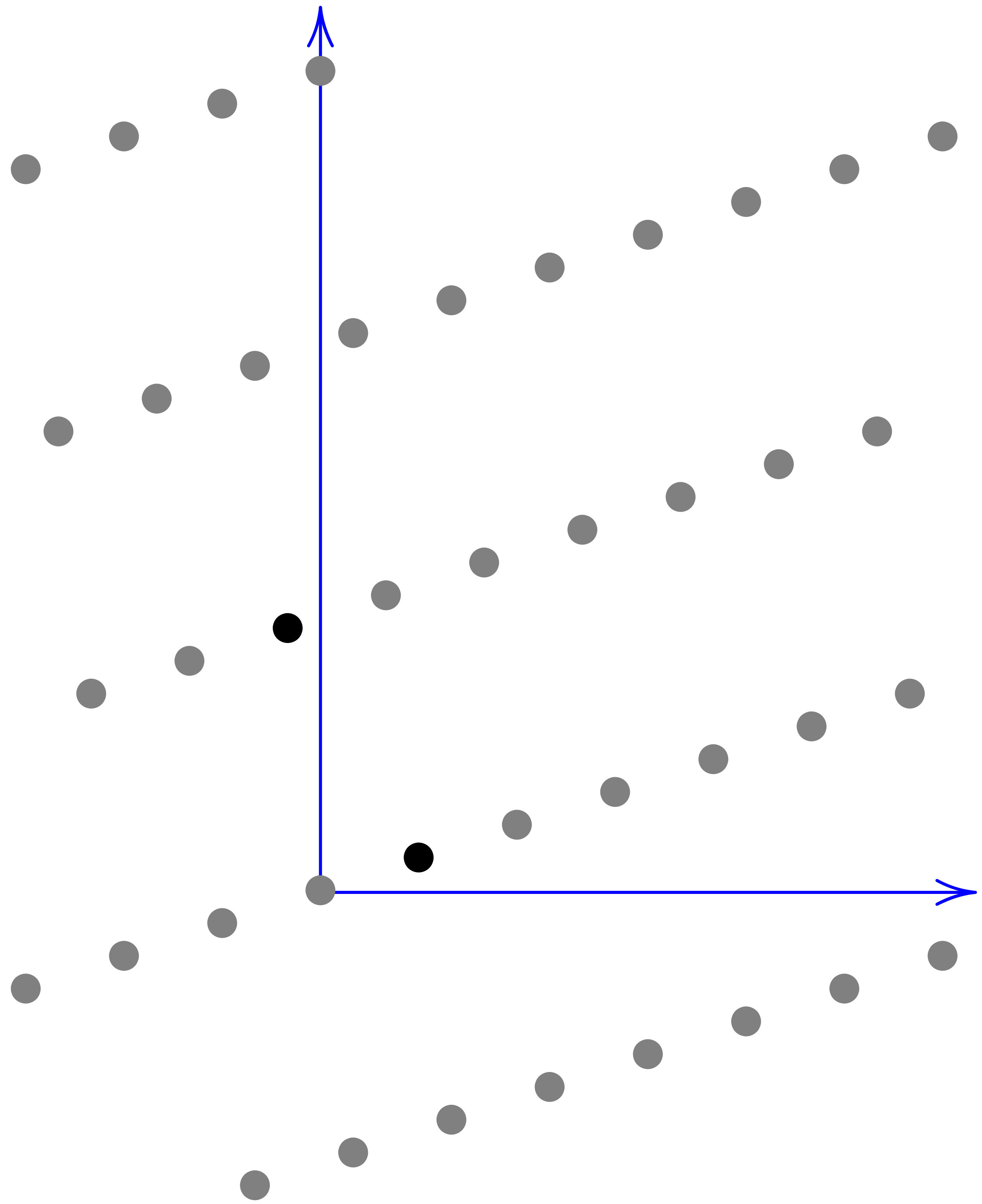

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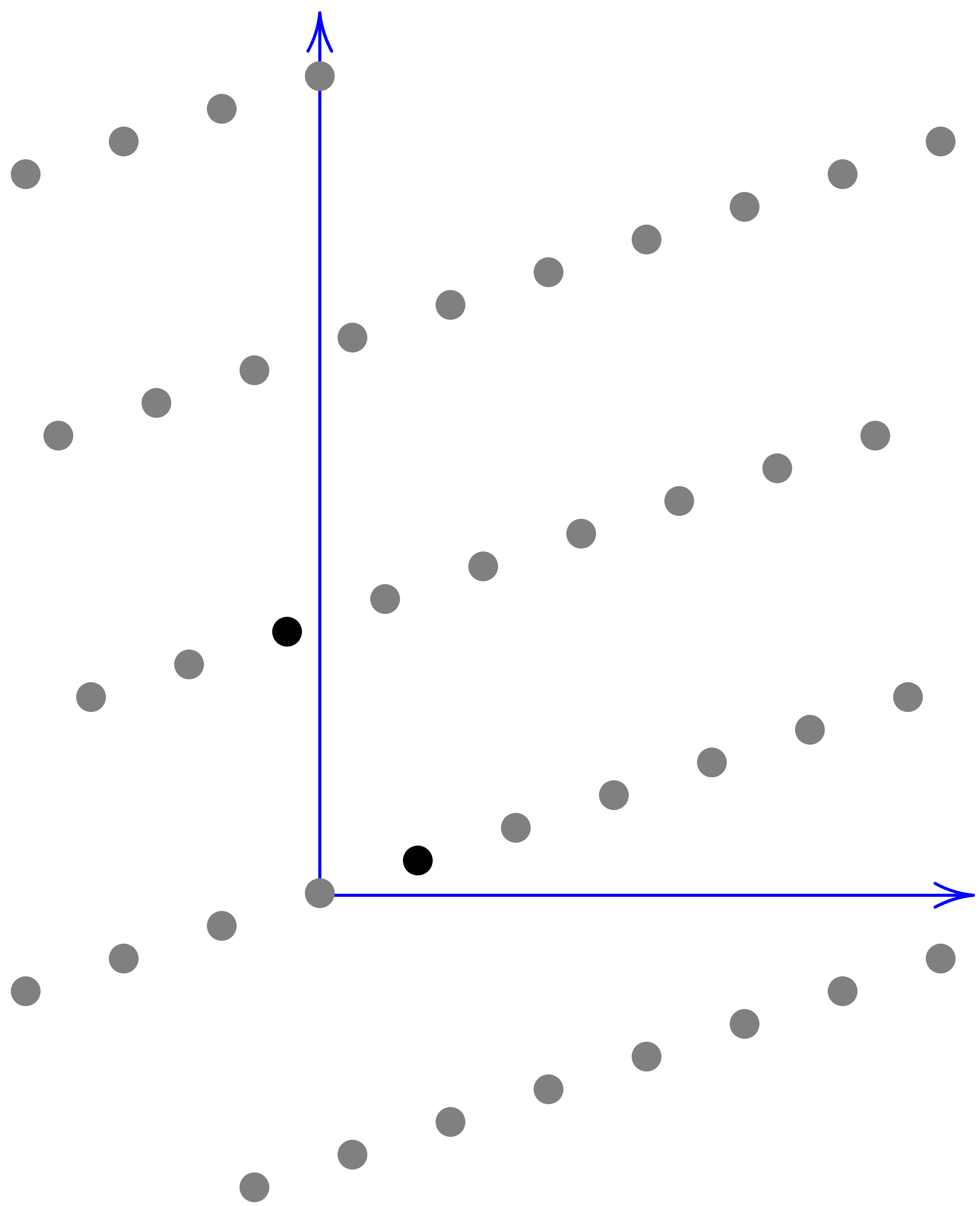

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What is the shortest nonzero vector in L?

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$(111, 1)$: shortest nonzero vector.
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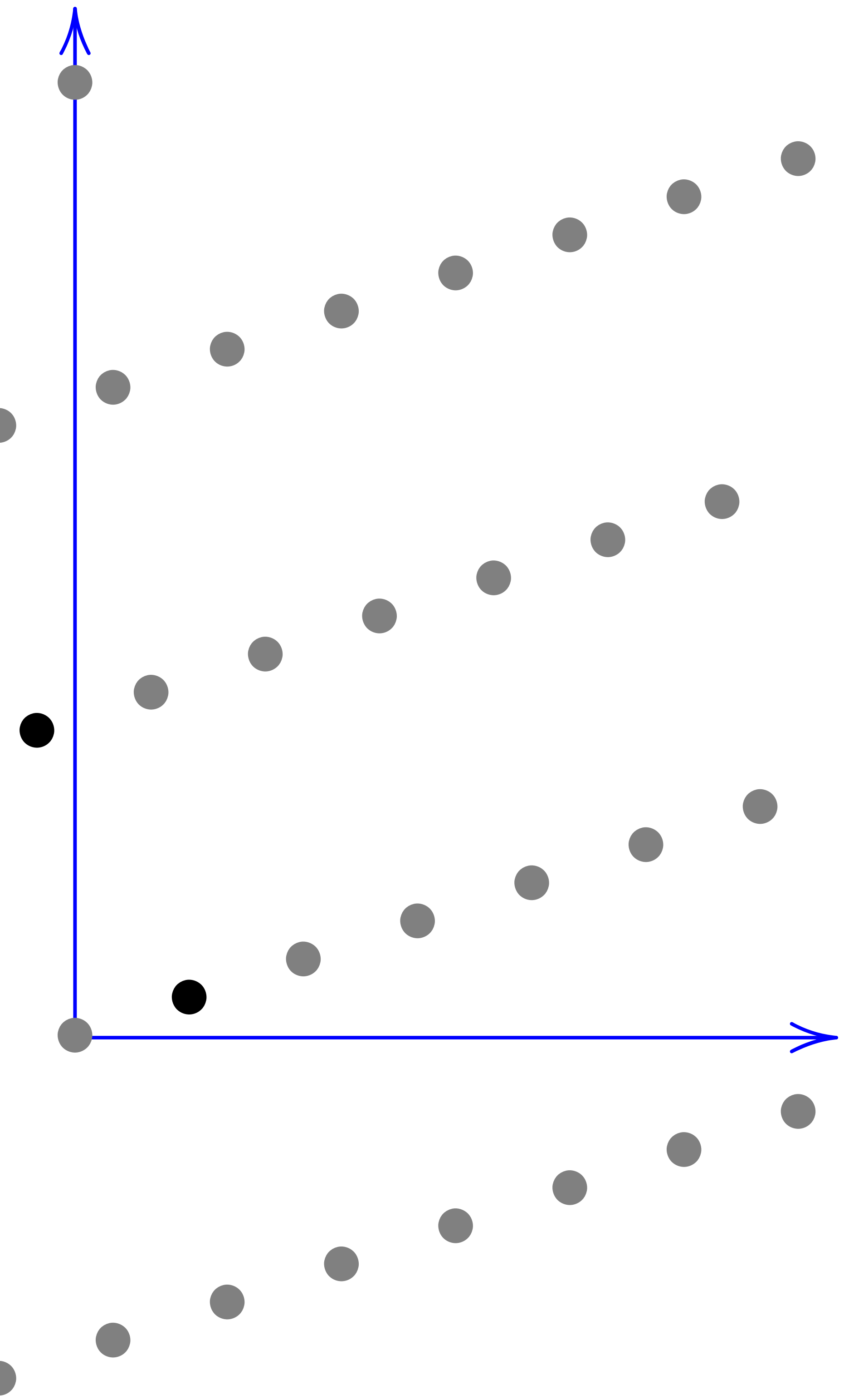

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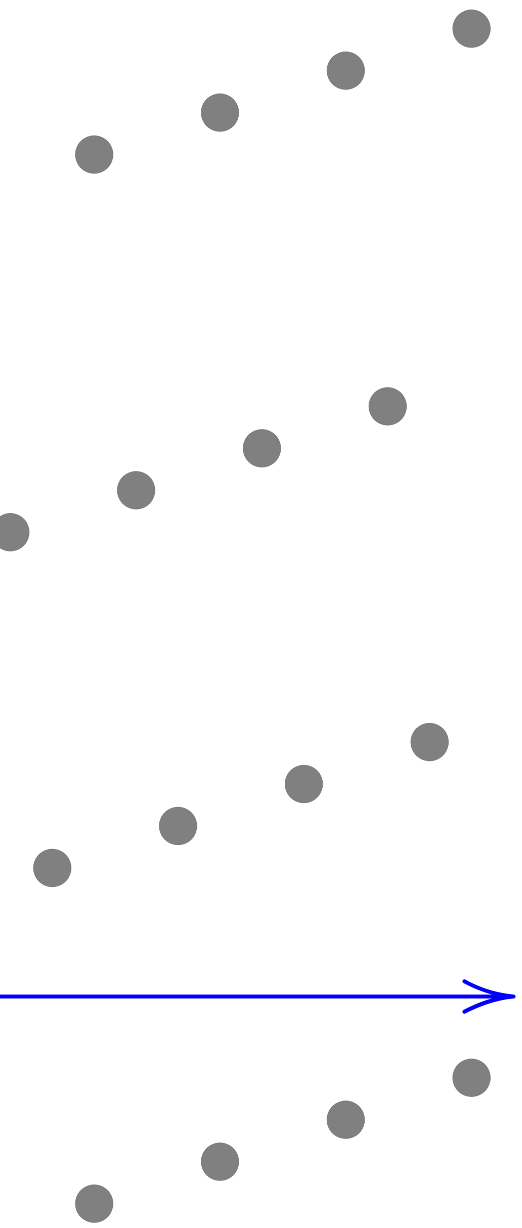

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Say j is minimal with $\deg r_j\sqrt{x} \le (\deg r_0)/2$. Then $\deg q_j \le (\deg r_0)/2$ so $\deg(q_j, r_j\sqrt{x}) \le (\deg r_0)/2$. Shortest nonzero vector.

$(q_{j+\epsilon}, r_{j+\epsilon}\sqrt{x})$ has degree $\deg r_0\sqrt{x} - \deg(q_j, r_j\sqrt{x})$ for some $\epsilon \in \{-1, 1\}$. Shortest independent vector.

field k, any r_0, r_1
$k[x]$ with $\deg r_0 > \deg r_1$:

Stevin computation:
$_2 = r_0 \bmod r_1$,
mod r_2, etc.

d: $q_0 = 0$; $q_1 = 1$;
$q_i - \lfloor r_i/r_{i+1} \rfloor q_{i+1}$.
$r_1 \equiv r_i \pmod{r_0}$.

view: Have
$)P + (1, r_1\sqrt{x})P =$
$)P + (q_{i+1}, r_{i+1}\sqrt{x})P$.

tinue until $r_{i+1} = 0$.
$_1\} = r_i / \operatorname{leadcoeff} r_i$.

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Find shortest nonzero
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Fact: $E/F = r_j/q_j$ so
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$\sum_i c_i/(x$
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so $(F, E$

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Proof: Assume

$\sum_i c_i/(x - a_i) = 0$ in $\mathbf{F}_{2^m}[x]/g$.

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4, $m = 10$, $t = 50$.
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1986 Niederreiter improvements:

Send Ke instead of $c + e$.

K is smaller than G whenever $mt < n - mt$. Compress K to $mt(n - mt)$ bits by requiring systematic form.

Does structure of Γ help attacker decrypt— e.g., compute $g, a_1, \ldots, a_n$?

All known "structural attacks" are much slower than information-set decoding. (Less conservative variants of McEliece encourage research.)

Does K leak more than Γ?

No with 1978 McEliece: matrix is explicitly randomized.

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Isn't this better than $n^2/\lg n$?

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plus some new improvements.

Gao and Mateer evaluate
$F = F_0 + F_1 x + \cdots + F_{n-1} x^{n-1}$
on a size-n $\mathbf{F}_2$-linear space.

Main idea: Write F as
$F_0(x^2 + x) + x F_1(x^2 + x)$.

Big overlap between $F(\alpha) =$
$F_0(\alpha^2 + \alpha) + \alpha F_1(\alpha^2 + \alpha)$
and $F(\alpha + 1) =$
$F_0(\alpha^2 + \alpha) + (\alpha + 1) F_1(\alpha^2 + \alpha)$.

"Twist" to ensure $1 \in$ space.
Then $\{\alpha^2 + \alpha\}$ is a
size-$(n/2)$ $\mathbf{F}_2$-linear space.
Apply same idea recursively.

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Too many variables for $m = 13$;
gcc ran out of memory.

Used `qhasm` register allocator
to optimize the variables.
Worked, but not very quickly.

Wrote faster register allocator.
Still excessive code size.

Built new interpreter,
allowing some code compression.
Still big; still some overhead.

Better solution:
stared at additive FFT,
wrote down transposition
with same loops etc.

Small code, no overhead.

Speedups of additive FFT
translate easily
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Further savings:
merged first stage with
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