Post-quantum cryptography

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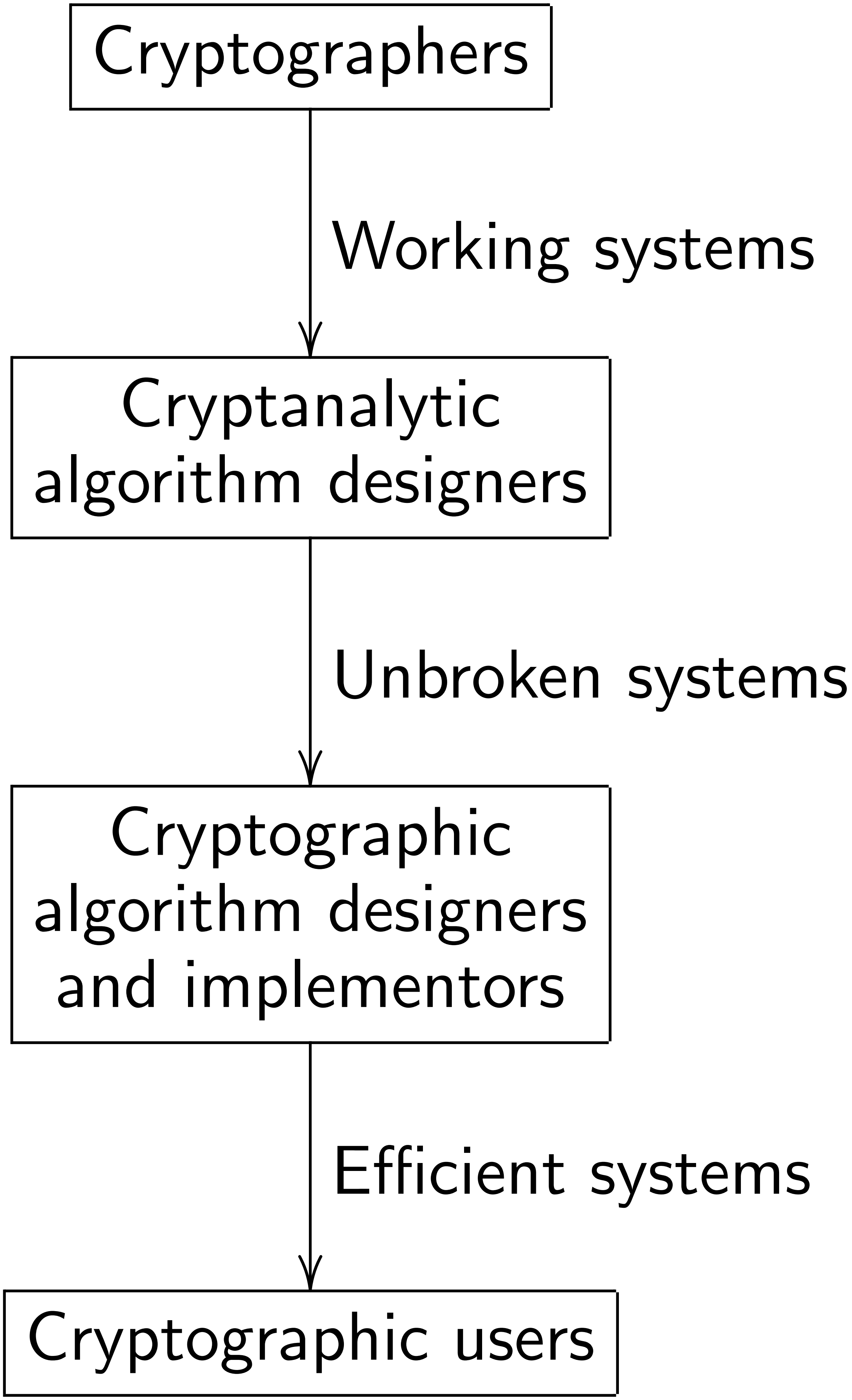
Cryptographers
Working systems
Cryptanalytic algorithm designers
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Cryptographic algorithm designers and implementors
Efficient systems
Cryptographic users

1. Working systems

Fundamental question for cryptographers:
How can we encrypt, decrypt, sign, verify, etc.?

Many answers:
DES, Triple DES, FEAL-4, AES, RSA, McEliece encryption, Merkle hash-tree signatures, Merkle–Hellman knapsack encryption, Buchmann–Williams class-group encryption, ECDSA, HFE^{v-}, NTRU, et al.

Detailed example
(not a very good cryptosystem!):
textbook exponent-3 RSA-1024.

Receiver's secret key: distinct 512-bit primes $p, q \in 2 + 3\mathbf{Z}$.

Receiver's public key: pq.

Sender's plaintext:
$m \in \{0, 1, \ldots, pq - 1\}$.

Sender's ciphertext: $m^3 \bmod pq$.

Receiver uses p, q to compute m given $m^3 \bmod pq$.

2. Unbroken systems

Fundamental question for *pre-quantum* cryptanalysts: What can an attacker do using $<2^b$ operations on a *classical* computer?

Fundamental question for *post-quantum* cryptanalysts: What can an attacker do using $<2^b$ operations on a *quantum* computer?

Goal: identify systems that are *not* breakable in $<2^b$ operations.

Examples of RSA cryptanalysis:

Schroeppel's "linear sieve", mentioned in 1978 RSA paper, factors pq into p, q using $(2+o(1))^{(\lg pq)^{1/2}(\lg \lg pq)^{1/2}}$ simple operations (conjecturally).

To push this beyond 2^b, must choose pq to have at least $(0.5+o(1))b^2/\lg b$ bits.

Note 1: $\lg = \log_2$.

Note 2: $o(1)$ says *nothing* about, e.g., $b = 128$.

1993 Buhler–Lenstra–Pomerance, generalizing 1988 Pollard "number-field sieve", factors pq into p, q using $(3.79\ldots + o(1))^{(\lg pq)^{1/3}(\lg \lg pq)^{2/3}}$ simple operations (conjecturally).

To push this beyond 2^b, must choose pq to have at least $(0.015\ldots + o(1))b^3/(\lg b)^2$ bits.

Subsequent improvements: $3.73\ldots$; details of $o(1)$.
But can reasonably conjecture that $2^{(\lg pq)^{1/3+o(1)}}$ is optimal —for classical computers.

Many "protocol" attacks.

e.g. attacker guesses user's m, verifies m^3 mod pq.

e.g. attacker hopes $m < (pq)^{1/3}$.

e.g. attacker sees how receiver reacts to $8m^3$ mod pq.

Typical fix: feed m through randomization+padding+"AONT".

"Simple RSA" (2001 Shoup): send r^3 mod pq for random r; use hash of r as AES-GCM key to encrypt and authenticate m.

Cryptographic systems surviving *pre-quantum* cryptanalysis:

Triple DES (for $b \le 112$),
AES-256 (for $b \le 256$),
RSA with $b^{3+o(1)}$-bit modulus,
McEliece with code length $b^{1+o(1)}$, Merkle signatures with "strong" $b^{1+o(1)}$-bit hash,
BW with "strong" $b^{2+o(1)}$-bit discriminant, ECDSA with "strong" $b^{1+o(1)}$-bit curve,
HFE^{v-} with $b^{1+o(1)}$ polynomials,
NTRU with $b^{1+o(1)}$ bits, et al.

Typical algorithmic tools for
pre-quantum cryptanalysts:
NFS, ρ, ISD, LLL, F4, XL, et al.

Post-quantum cryptanalysts
have all the same tools
plus quantum algorithms.

Spectacular example:
1994 Shor factors pq into p, q
using $(\lg pq)^{2+o(1)}$
simple quantum operations.
To push this beyond 2^b,
must choose pq to have at least
$2^{(0.5+o(1))b}$ bits. Yikes.

Cryptographic systems surviving *post-quantum* cryptanalysis:

AES-256 (for $b \le 128$),
McEliece code-based encryption with code length $b^{1+o(1)}$,
Merkle hash-based signatures with "strong" $b^{1+o(1)}$-bit hash,
$\text{HFE}^{\text{v}-}$ MQ signatures with $b^{1+o(1)}$ polynomials,
NTRU lattice-based encryption with $b^{1+o(1)}$ bits,
et al.

3. Efficient systems

Fundamental question for designers and implementors of cryptographic algorithms: Exactly how efficient are the unbroken cryptosystems?

Many goals: minimize encryption time, size, decryption time, etc.

Pre-quantum example:
ECDSA with "strong" $b^{1+o(1)}$-bit curve verifies signature in $b^{2+o(1)}$ simple operations.
Signature occupies $b^{1+o(1)}$ bits.

Users have cost constraints.

Cryptographers, cryptanalysts, implementors, etc. tend to focus on RSA and ECC, citing these cost constraints.

But we think that the most efficient unbroken *post-quantum* systems will be hash-based systems, code-based systems, lattice-based systems, multivariate-quadratic systems.