Two completely unrelated topics:
(1) McBits;
(2) Post-Quantum RSA

D. J. Bernstein
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Thanks for (1) to:
Cisco University Research Program

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Bonus topic added today:

0. Wild McEliece (joint work with Tanja Lange, Christiane Peters)

Conventional wisdom on McEliece using degree-t Goppa: t errors over $\mathbf{F}_2$, but only $t/2$ errors over $\mathbf{F}_q$ if $q > 2$.

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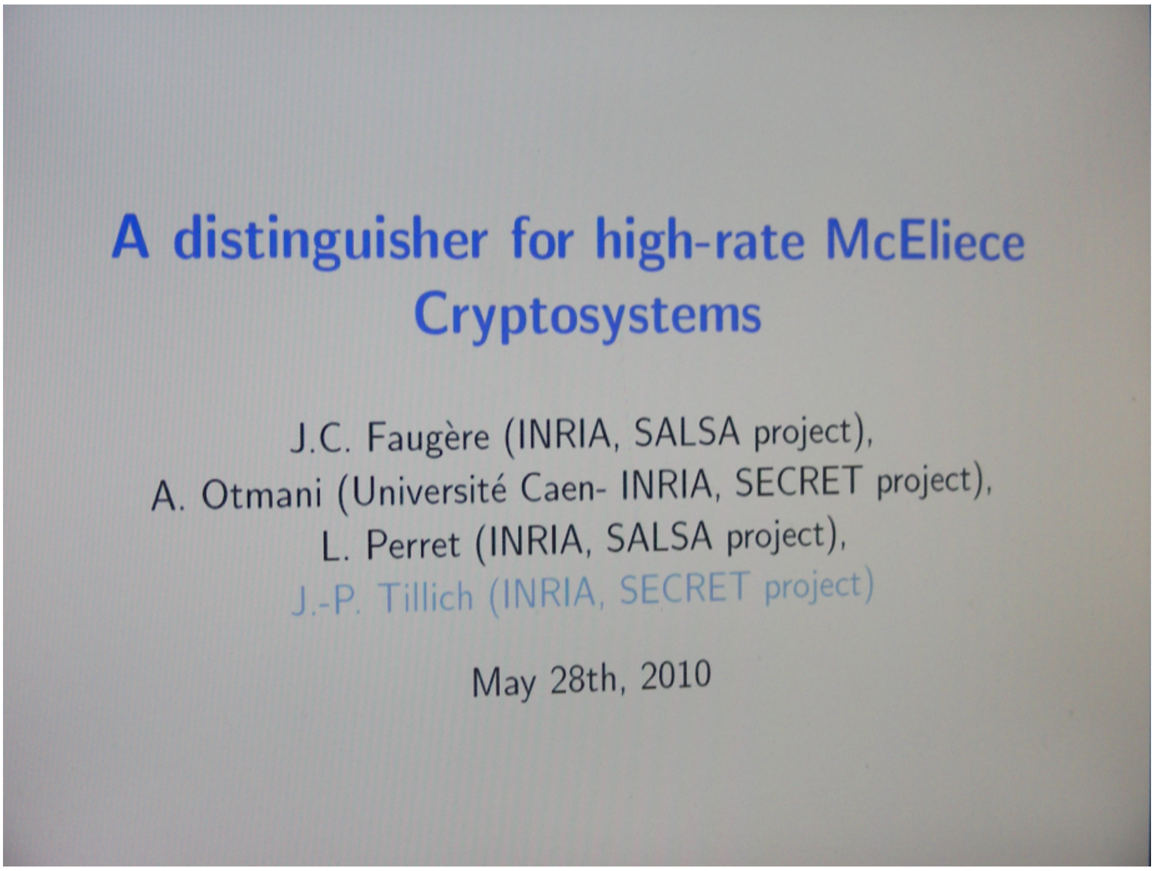
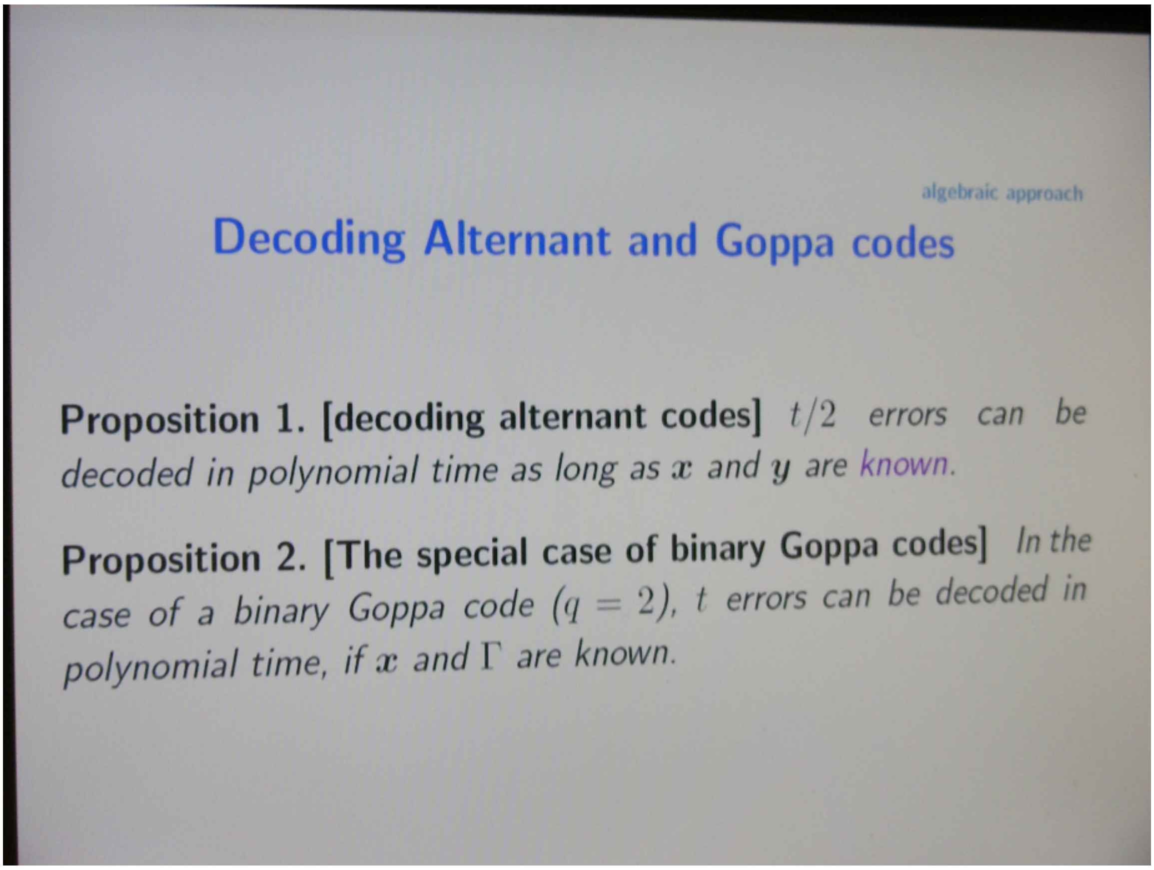


Decoding Alternant and Goppa codes

Proposition 1. [decoding alternant codes] *$t/2$ errors can be decoded in polynomial time as long as x and y are known.*

Proposition 2. [The special case of binary Goppa codes] *In the case of a binary Goppa code ($q = 2$), t errors can be decoded in polynomial time, if x and Γ are known.*



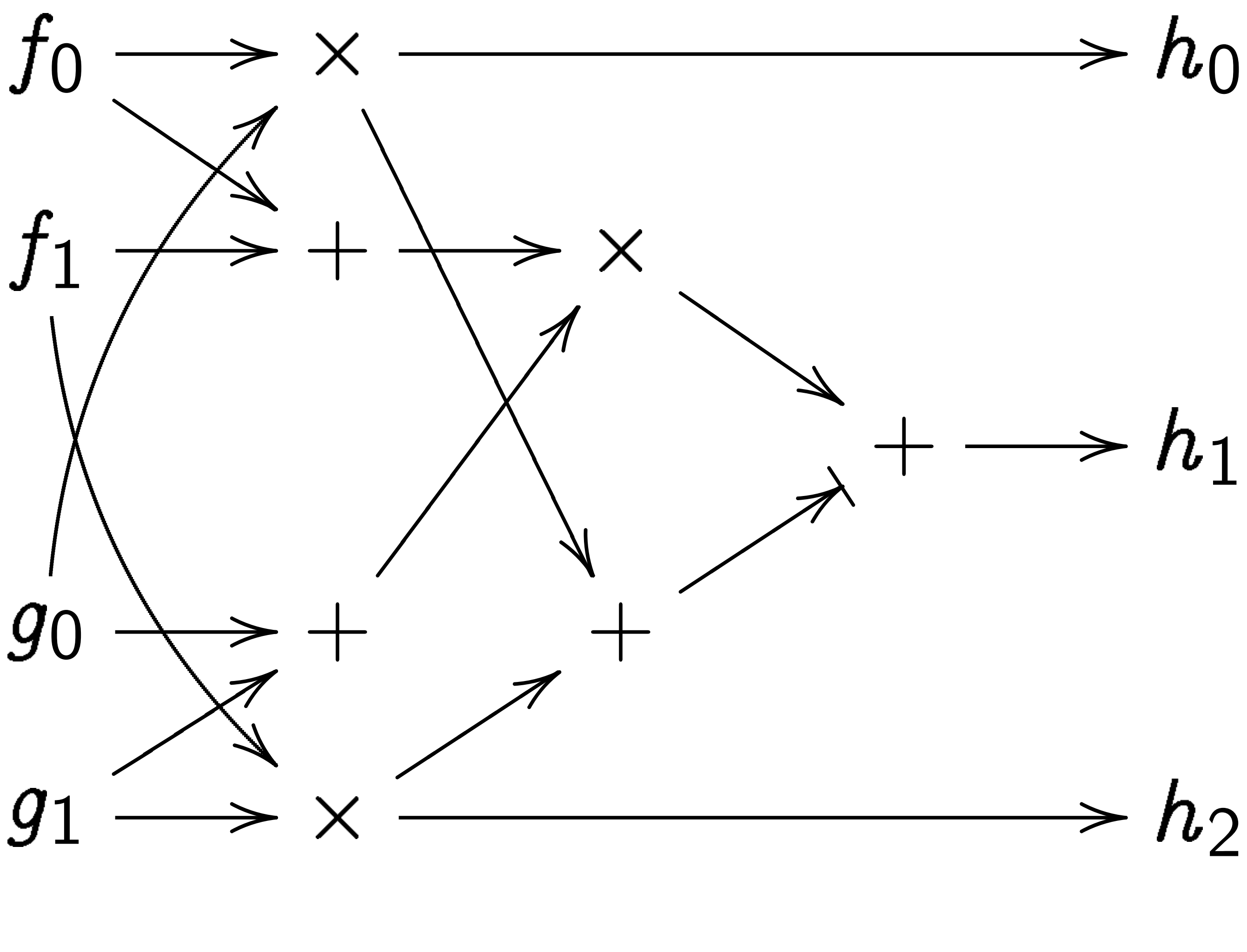
New: "Wild McEliece" uses $qt/(2(q-1))$ errors over $\mathbf{F}_q$.

More details: See talk from C. Peters from two days ago.

1. McBits: Arithmetic circuits for code-based cryptography

An $\mathbf{F}_2$-arithmetic circuit starts from inputs and constants and computes a chain of two-input $\mathbf{F}_2$-adds $u, v \mapsto u + v$, two-input $\mathbf{F}_2$-mults $u, v \mapsto uv$.

Example, not the smallest 2×2 polynomial multiplier:



What I'm working on:
fast arithmetic circuits
for confidence-inspiring
code-based public-key encryption.

Circuits are good for security:
no conditional jumps;
no variable array indices;
no input-dependent timings;
no software side channels.

Plan to publish software
and place into public domain.

Main challenge: Speed.

Metric for this project:
"ops" $=$ #adds $+$ #mults.

Clear definition; simple.

Not a bad predictor of
bitsliced software speed.

Also not a bad predictor of
throughput of unrolled hardware.

Warnings: metric doesn't see
code size ("ops" unrolls loops),
communication costs, etc.

Counting bit operations
rewards fast mult algorithms,
as in new ECC speed records
(2009 “batch binary Edwards”).
Now exploring Gao–Mateer mult.

Use fast multipoint evaluation to
eliminate conditional jumps from
fast root-finding; $n^{1+o(1)}$ ops.

Most annoying part to write:
$n^{1+o(1)}$ fast continued fraction
without conditional jumps.

Biggest asymptotic bottleneck:
matrix randomizer, $n^{2+o(1)}$ ops.
Can reduce 2 with more batching.

2. Post-Quantum RSA:
Is it possible that
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Is this actually true?

Some methods to factor n
(assuming standard conjectures):

Trial division finds p
using $(p + \lg n)^{1+o(1)}$ bit ops.

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Number-field sieve finds p using $(2^{c(\lg n)^{1/3}(\lg \lg n)^{2/3}})^{1+o(1)}$ bit ops.

Shor's algorithm finds p
using $(\lg n)^{2+o(1)}$ qubit ops.

Let's assume that qubit ops
aren't much harder than bit ops,
and that $o(1)$ isn't very big.

Does Shor supersede NFS?
Yes.

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Does Shor supersede ECM?
Not necessarily!

ECM beats Shor for small p: compare $2 \lg p \lg \lg p$ to $(\lg \lg n)^2$.
Best small-p algorithm I know: GEECM.

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compute cube root mod $n = pq$
by computing and combining
cube roots mod p and q.
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1997/1998 Tandem patent

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Use "multi-prime RSA."
1997/1998 Tandem patent
but already in 1983 RSA patent:
"the present invention
may use a modulus n which is
a product of three or more primes
(not necessarily distinct)."

Public key $n = p_1 p_2 \cdots p_k$.
Secret primes $p_1, p_2, \ldots, p_k$ with
$\lg p_i \in b^{2+o(1)}$, $k \in 2^{(1+o(1))b/2}$.

Key: $2^{(1+o(1))b/2}$ bits.
Encryption: $2^{(1+o(1))b/2}$ bit ops.
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Concrete analysis suggests that
RSA with 2^{31} 4096-bit primes
provides $> 2^{100}$ security
vs. all known quantum attacks.
Key almost fits on a hard drive.